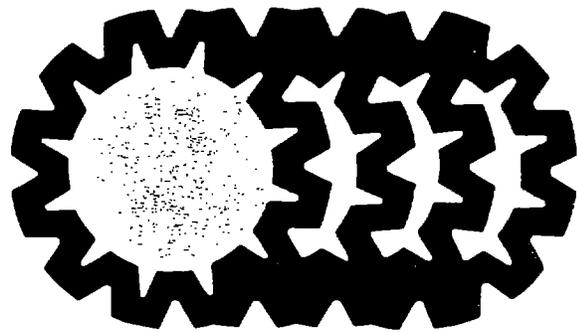


NIOSH

TECHNICAL INFORMATION

***Statistical Methods for the
Determination of Noncompliance with
Occupational Health Standards***



STATISTICAL METHODS FOR THE DETERMINATION
OF NONCOMPLIANCE WITH OCCUPATIONAL HEALTH STANDARDS

Nelson A. Leidel

and

Kenneth A. Busch

U.S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
Public Health Service
Center for Disease Control
National Institute for Occupational Safety and Health
Division of Laboratories and Criteria Development
Cincinnati, Ohio 45202

April 1975

For sale by the Superintendent of Documents, U.S. Government
Printing Office, Washington, D.C. 20402

Mention of company or product names is not to
be considered as an endorsement by NIOSH.

(Formerly NIOSH TR-76)

HEW Publication No.(NIOSH) 75-159

PREFACE

The first NIOSH recommendations concerning statistical methods for the determination of compliance with an occupational health standard appeared in 1972 in the NIOSH Criteria Document "Occupational Exposure to Carbon Monoxide," HSM 73-11000 (pp. VIII-2 and 3). Subsequent research and an analysis of existing literature demonstrated that the assumptions made for the above method were inappropriate. Since 1972, research work has continued in NIOSH, both in-house and contracts, with the goal of developing practical statistical methodology which can be applied to the problems of sampling strategy and decision making in regard to occupational health employee exposure standards.

This report presents the first results of NIOSH work on predictive and analytical statistical methods in the field of industrial hygiene. Other reports will follow with more extensive presentations of techniques and statistical theory. This technical report is concerned solely with noncompliance statistics and is oriented toward the governmental compliance officer. However, the conclusions regarding sampling strategy are equally applicable to employers and industry industrial hygienists.

ACKNOWLEDGEMENTS

The authors wish to gratefully acknowledge the many reviews, suggestions, and constructive criticisms that contributed to this report. Special thanks are due Jeremiah Lynch, Bill Kelley, and Paul Roper for their extensive reviews of the many versions of this work. Additional thanks are due Chuck Atkins, Jeff Burton, Kathy Carlberg, Lorice Ede, Dave Huebner, Dick Kupel, Lee Larsen, Gus Lauman, Chuck McCammon, Ann Saalwaechter, and Jay Chmiel for their contributions. John Bryant and Dr. Herb Stokinger provided much encouragement and motivation to the authors toward the completion of this report. Finally, very special thanks are due Myra Brooks, Mary Geimeier, and Pat Zehnder for their patient typing of the drafts of this report.

TABLE OF CONTENTS

Abstract	vii
Nonmenclature	viii
Introduction	1
Normal and Log-normal Frequency Distributions	3
Compliance vs. Noncompliance Statistics	5
Sampling Strategy	5
Sample Duration	6
Number of Samples	8
Sampling Periods	10
Ceiling Standards	12
Treatment of Gas Detector Tube Data	13
Procedures for Data Analysis	14
Full-Period Single-Sample Procedure	16
Full-Period Consecutive Samples Procedure	18
Partial-Period Consecutive Sample(s) Procedure	19
Grab Samples Data Analysis	20
Examples of Procedures	
Example #1 for Full-Period Consecutive Samples Procedure . . .	21
Example #2 for Full-Period (Ceiling) and Partial Period Consecutive Samples Procedures	22
Example #3 for Full-Period Consecutive Samples Procedure . . .	25
References	27
Figures	31

TABLE OF CONTENTS (CONT.)

Appendix A - Grab Samples Procedure	39
0.0 - Nomenclature	40
1.0 - Introduction	41
2.0 - Description of procedure	42
3.0 - Examples of procedure	43
4.0 - Decision chart and estimation nomographs	52

ABSTRACT

Procedures are presented for determining if the 95% Lower Confidence Limit of the concentration of a contaminant in an occupational environmental air sample or of the arithmetic average for a group of such samples exceeds an occupational health standard. The determination can be made with a known maximum probability of making an incorrect decision that a state of noncompliance exists. Recommendations are given concerning the duration of samples, the number of samples to take and the period(s) during the work day when the samples should be collected. The advantages of a full-period sample (or consecutive samples) over the mean of several grab samples are discussed.

When the true arithmetic average air concentration is estimated from the mean of a group of random grab samples the log-normal distribution is assumed for the individual grab samples. When only one sample is examined or when one or several samples are taken for the entire time period for which a standard is defined the normal distribution is assumed. In the latter cases, variability over time is absent, but sampling and analytical errors are present.

NOMENCLATURE

- C.V. coefficient of variation, a measure of relative dispersion, also known as the relative standard deviation = s/\bar{x}
- GM geometric mean = $(10)^{\bar{x}_\ell}$
- GSD geometric standard deviation = $(10)^{s_\ell}$
- LCL lower confidence limit (95% one-sided) of a data value or of the arithmetic mean of a group of normally or lognormally distributed samples
- n number of measurements being analyzed
- s standard deviation of original data
- s_ℓ standard deviation of \log_{10} of original data
- std occupational health employee exposure standard such as the federal standards in 29 CFR 1910.93
- TWA time weighted average employee exposure as defined in 29 CFR 1910.93(d)(1)
- x_i a single air concentration measurement from a group of such measurements
- T_i time duration of a single sample
- T total time duration for a group of n samples
- \bar{x} arithmetic mean of a group of air concentration measurements x_i
- \bar{x}_ℓ arithmetic mean of the \log_{10} of the original data

- σ standard deviation of a sampling/analytical method which is well known from prior data
- $\sigma_{\bar{x}}$ standard deviation of \bar{x}
- Σx sum of a group of air concentration measurements over all n values

INTRODUCTION

The objective of this paper is to present sound statistical procedures for the collection and evaluation of sample results to determine if a state of noncompliance with an occupational health standard exists. We will present procedures for calculating the 95% Lower Confidence Limit (LCL) of an occupational environmental sample or the arithmetic average of a group of such samples. One may then compare the results of occupational environmental sampling to an occupational health standard and make a decision with a known chance of making an incorrect decision that a state of noncompliance exists.

The occupational health standard may be either an 8-hour time-weighted average (TWA) or ceiling standard defined for a short time interval (generally 30 minutes or less). It should be emphasized that numerical calculations are necessary only if the sample mean is greater than the standard.

This technical report was written primarily to serve as a handbook for governmental industrial hygienists and compliance officers, both state and federal, who are responsible for making decisions in regard to noncompliance with occupational health standards. To accomplish this goal the report contains many examples of the recommended procedures. Also several nomograms are presented to speed calculation of results if a desk calculator is not available.

The report treats only the statistical analysis of occupational environmental data. Topics such as choice of sampling location, mobility of the worker, variation in work procedures and processes, appropriate application of the standard to the process being sampled, and finally appropriate selection of breathing zone, personal and/or area samples are not treated in this report. In all cases one must avoid the trap of falling into a "numbers game" and keep in proper perspective what the data represents in relation to what the worker is exposed to.

Roach (1,2,3) recognized the fact that a small number of occupational environmental samples can only yield an estimate of the true mean air concentration of the substance sampled. He pointed out that, due to the uncertainty in the mean value obtained from only a few samples, allowance must be made for this uncertainty when comparing the mean with the occupational health standard in determining whether a condition of non-compliance exists.

However, Roach and others have made the mistake of assuming that all occupational environmental data was best described by the normal distribution. Recent work has shown that occupational environmental data is usually better described by fitting a log-normal distribution to the measurements. But random sampling and analytical errors tend to be more nearly normally

distributed so that this distribution can be used when only one sample is examined or when a sample or several samples are taken for the entire time period for which the standard is defined.

In all cases the arithmetic mean of the data is the best descriptor of the average concentration to which a man is exposed. The variability of the data is accounted for by calculating the 95% Lower Confidence Limit (LCL) on the sample or arithmetic mean of a group of samples. The log-normal distribution is assumed if the samples are "grab" samples (several samples taken for a small fraction of the time period for which the standard is defined).

The statistical procedures presented below will not detect and do not allow for analysis of highly inaccurate results, i.e., systematic (nonrandom) errors or mistakes. The detection and elimination of mistakes is primarily a technical rather than a statistical problem. To assure accurate results one must have an instrument calibration program and a quality control program for laboratory analysis. Systematic errors must also be known ahead of time whether from the instrument calibration procedure or the laboratory quality control program. If a constant systematic error is known to exist in an instrument or analytical procedure then correct the sample mean of the data before analyzing for noncompliance.

NORMAL AND LOG-NORMAL FREQUENCY DISTRIBUTIONS

Before sample data can be statistically analyzed we must have knowledge of the frequency distribution of the results or some assumptions must be made. Roach (1,2,3) and Kerr (4) have assumed that environmental data is normally distributed. ~~However, it is well established (5,6,7,8) that most community air pollution environmental data is better described by a log-normal distribution.~~ That is, the logarithms (either base e or base 10) of the data are approximately normally distributed. Most importantly, Breslin, et al (9), Sherwood (10,11), Jones and Brief (12), Gale (13,14), Coenen (15,16), Hounam (17), and Juda and Budzinski (18,19) have shown that occupational environmental data from both open air and confined work spaces for both short (seconds) and long (days) time periods are log-normally distributed.

What are the differences between normal and log-normally distributed data? First, it should be remembered that a "normal" distribution is completely determined by two parameters: 1) the arithmetic mean (μ) and 2) the standard deviation (σ) of the distribution. On the other hand, a "log-normal" distribution is completely determined by 1) the median or geometric mean (GM) and 2) the geometric standard deviation (GSD). For log-normally distributed data, a logarithmic transformation of the original data is normally distributed. The GM and GSD of the log-normal distribution are merely antilogs of the mean and standard deviation of the logarithmic transformation. Normally distributed data has a symmetrical distribution curve while log-normally distributed environmental data is generally positively skewed (long "tail" to the right indicating a larger probability of very large concentrations than for normally distributed data). Figure 1 compares a log-normal distribution to a normal distribution that has the same arithmetic mean (μ) and standard deviation (σ).

The conditions conducive to (but not all necessary for) the occurrence of log-normal distributions are found in occupational environmental data. These conditions are (15):

- 1) the concentrations cover a wide range of values, often several orders of magnitude,
- 2) the concentrations lie close to a physical limit (zero concentration),
- 3) the variability of the measured concentration is of the order of the size of the measured concentration and,
- 4) there is a finite probability of very large values (or data "spikes") occurring

The variability of occupational environmental data (differences between repeated measurements at the same site) can usually be broken into three major components: 1) random errors of the sampling method, 2) random errors of the analytical method and 3) variability of the environment with time. The first two components of the variability are usually known in advance and are approximately normally distributed. However, the environ-

mental fluctuations of a contaminant in a plant usually greatly exceed the variability of known instruments (often by factors of 10 or 20). The above components of variability were discussed in an article by LeClare, et. al. (20).

When several samples are taken in a plant to determine the average concentration of the contaminant and estimate the average exposure of an employee then the log-normal distribution should be assumed. However, ~~the normal distribution may be used in the special cases of 1) taking a sample to check compliance with a ceiling standard and 2) when a sample (or samples) is taken for the entire time period for which the standard is defined (be it 15 minutes or eight hours).~~ In these cases the entire time interval of interest is represented in the sample and only sampling and analytical errors are present.

Both Coenen (15,16) and Sherwood (11) state that the the arithmetic mean of the data must be used in the assessment of measurement results because the arithmetic mean is the best descriptor of the average concentration to which a man is exposed. However, when the arithmetic mean (\bar{x}) for a group of samples is compared to a standard to determine if a state of noncompliance exists the variability of the data must be taken into account. This can be done by calculating the 95% Lower Confidence Limit (LCL) on the arithmetic mean of the group of samples whether the samples are normally or log-normally distributed. In order to statistically demonstrate a condition of non-compliance the LCL must exceed the standard as shown in Figure 2. If the LCL is below the standard then noncompliance cannot be shown, but compliance is not necessarily proven. More will be said about this later. Of course, if (\bar{x}) itself is below the standard, it is unnecessary to calculate the LCL or perform any formal statistical test.

Both Coenen (15) and Juda and Budzinski (18,19) have given equations for confidence limits on the arithmetic mean of a log-normal distribution. However in both cases simplifying assumptions had to be made. Coenen's (15) assumption concerning the distribution of individual errors seemed to have questionable statistical basis. Juda and Budzinski (18,19) equated the arithmetic mean and the geometric mean in order to arrive at approximate confidence limits.

Since both of the above approaches utilized questionable approximations it was decided to use only rigorous statistical theory. A NIOSH contractor, Systems Control, Inc. has developed a procedure that uses a decision chart rather than calculating the LCL (21, 22). The procedure uses original data normalized to the standard and the variability of the actual data. This procedure is presented as Appendix A of this report.

COMPLIANCE VS. NONCOMPLIANCE STATISTICS

The one-sided statistical tests given in this report are only for the purpose of determining if a condition of noncompliance exists. If the test yields a non-significant result, one cannot conclude that a condition of compliance exists, but rather, only that insufficient evidence for non-compliance is available. Thus, there are two alternatives in the case of a nonsignificant result. First, a condition of compliance might exist. But there is also the possibility that enough measurements are not available to definitively answer the question one way or the other. Additional statistical theory not presented here must be used to evaluate the latter two possibilities.

Simply stated, statistics for noncompliance are oriented toward the governmental compliance officer and are used to make a decision on data that occurred on a particular day. The statistical tests are designed to limit the maximum "risk" (probability) to 5% of making a wrong decision. i.e. the risk of deciding for noncompliance when in fact the process sampled is in compliance. Leidel (23) has discussed the selection of an appropriate level of risk for these type statistical tests and the interaction between the protection levels afforded the employer and employee.

This technical report is concerned solely with such noncompliance statistics. Conversely, statistics for compliance are oriented toward the employer or industrial hygiene consultant to a company. The National Institute for Occupational Safety and Health currently has underway a contract with Systems Control, Inc. of Palo Alto, Ca. regarding "Sequential Sampling Plans and Decision Theory for Employer Monitoring of Employee Exposures to Industrial Atmospheres" (CDC-99-74-75). The contractor is developing a manual that adapts appropriate sampling/decision theory into a simplified procedure which can be used by non-statistically trained employers or their representatives. This manual should be available in the summer of 1975.

However, there already exists in the literature several articles on compliance statistics. Jones and Brief (12) give an excellent discussion of how to plot log-normal occupational environmental data and interpret the resulting graph. Sherwood (11) and Coenen (15) also discuss how to interpret sample data from a log-normal distribution.

SAMPLING STRATEGY

In trying to assess an occupational environment and estimate employee exposures three important questions must first be answered concerning the sampling strategy. Over how long a period should each sample be taken (sample duration)? How many samples should we take? Finally, at what times during the day should we take the samples?

Sample Duration

For some grab sample devices the sampling period is predetermined. Colorimetric detector tubes are an example. There is a direct reading dust monitor that samples for a fixed one minute period. Some substances such as asbestos (24, 25) may require certain sampling times to attain the best results during the analytical procedure. In many cases though we have a choice of a wide range of sampling times from a few seconds to eight hours. Intuitively we might feel that lengthening the sample period would significantly increase the certainty of the information we collect. That is we would say that an increase in sample time would lower the variability of the data and reduce the width of the confidence limits on the mean, thereby yielding a "better" answer. However, the above assumption is almost completely false for short term grab samples.

Coenen (16) derived a relation that showed the reduction in dispersion (corresponding to the variance of Gaussian random processes) as a function of increase in sampling time. He concluded that data variability (dispersion) decreases only slightly with sampling time and that an increase in sampling time even by a factor of ten is usually not worthwhile. Using Coenen's relation Table 1 was developed, assuming typical autocorrelation coefficients of 0.9 and 0.7 between consecutive samples of length equal to the basic period.

Table 1 - Possible % reduction in standard deviation of individual short-term grab samples with increasing sampling durations

<u>multiple of basic sampling period</u>	<u>possible % reduction in standard deviation</u>
2	3 to 10%
5	8 to 21%
10	11 to 29%

The overall sample variance is the sum of a) the sampling method variance, b) the analytical method variance, and c) the environmental concentration variance. The data of Larsen, et. al. (26) showed percentage reductions in geometric standard deviations (GSD) similar to those in Table 1. LeClare, et. al. (20) found similar reductions and concluded that the small reductions obtained in the longer term sample variances were due to the limiting effect of the analytical method variance. Therefore the primary consideration in selecting short sampling times should be the analytical method. Each analytical method requires a minimum amount of material. This should be known in advance and the sample period selected accordingly. Any increase in the sampling period past the minimum time required to collect an adequate amount of material is unnecessary and unproductive.

Saltzman (27) concluded that the choice of sampling time is ". . . an important decision affecting the results." This is true if we are trying to gather detailed information of the true component fluctuations in air concentrations. However when attempting to make a decision on a possible noncompliance situation it is better to take shorter samples because this allows us to take more samples in a given day. We shall see that it is much more important to collect several samples of short duration than to collect one medium length sample covering the same total sampling period. This is true because the random analytical/sampling errors can be "averaged out" (along with the longer-term environmental fluctuations during the sampling day) by taking a mean of random independent short samples.

The limiting case of the above occurs if we have the ability to sample for nearly 100% of the time period for which the standard is defined. If we can sample for the entire length of the standard's averaging period or close to it (with either one or several consecutive samples) we can obtain a better estimate of the true average employee exposure. This is because we only have to contend with the sampling/analytical errors which are much smaller than the environmental fluctuations which affect short term samples. Thus, there is a marked advantage in using a single full-period sample (or consecutive samples over the full period) when attempting to demonstrate noncompliance. It is much more difficult to demonstrate noncompliance using the mean of several grab samples because the additional variability due to the environmental fluctuations lowers the LCL. This is shown in Figure 3 which demonstrates the effect of the number of grab samples on requirements for demonstration of noncompliance.

The family of three curves are for various levels of data variability as measured by geometric standard deviations (GSD) ranging from 1.5 to 3.0 ($s_p = 0.176$ to 0.477). Fortunately much industrial hygiene data have GSDs of 1.5 to 2.5. For this range of GSDs and sample sizes of 3 to 10, one must generally obtain a measured average exposure of 1.2 to 2.5 times the standard in order to demonstrate noncompliance. For GSDs greater than 3.0 it is very difficult to show noncompliance because the observed mean exposure has to be so far above the standard. Figure 3 is based on the procedure presented in Appendix A.

However, if we use a full-period sample (or consecutive samples) in attempting to demonstrate noncompliance, the amount the observed mean must be above the standard is much lower. Figure 4 shows the effect of consecutive full-period sample size on requirements for demonstration of noncompliance. The five curves are for various sampling/analytical coefficients of variation (CV) ranging from 0.05 to 0.25. See Table 2 for typical CVs. For the range of typical CVs and sample sizes of 1 to 4, one must have an average concentration of 1.05 to 1.41 times the standard in order to demonstrate noncompliance. Suppose we are able to take four consecutive 2-hour samples for asbestos. The Full-Period Consecutive Samples procedure (and Figure 4) shows that for asbestos (CV = 0.22, Table 2) the mean of the four samples has to be only 18% above the standard to show noncompliance. Single asbestos samples taken for comparison with the ceiling standard have to be 36% above the standard to show noncompliance.

Number of Samples

The question of how many samples to take is vital since it relates directly to the confidence we have in our resulting estimate of the mean air concentration and employee exposure. We see the effect of sample size on requirements for demonstration of noncompliance in Figures 3 and 4. For grab samples (Figures 3) the curves for GSDs of 1.5 to 2.5 change relatively slowly after sample sizes of 7 to 8. For full-period consecutive samples (Figure 4) a similar leveling off occurs of the curves relating the value of $\bar{x}/\text{std.}$ required to demonstrate noncompliance to sample size.

For full-period consecutive samples, Figure 4 shows that based on statistical considerations alone, a suitable number of samples is around four to seven. However, practicality and costs of sampling and analysis must be considered. Most long duration sampling methods cannot be run for longer than about 4 hours per sample. Thus most full-period consecutive sampling strategies would obtain at least two samples when an 8-hour average standard is sampled for. If one had a sampling/analytical technique with a C.V. of 10%, Figure 4 shows that the $\bar{x}/\text{std.}$ required to demonstrate noncompliance decreases from about 1.12 for 2 samples to about 1.06 for 7 samples. Or for 2 samples we can demonstrate noncompliance when the mean of the 2 samples is 12% above the standard. But with 7 samples we can demonstrate noncompliance when the mean of 7 samples is 6% above the standard. Above seven samples the small decrease obtained in variability is not normally justified when compared to the time and effort required to obtain additional samples. There are theoretical benefits to be gained from larger sample sizes, but in terms of the disproportionately large additional costs incurred (especially analytical) the benefits are usually negligible. Thus on a cost benefit basis we can conclude that 2 consecutive full-period samples (about 4-hours each for an 8-hour average standard) is the "best" number to take.

For grab samples, less than 4 samples lead to unreasonably large $\bar{x}/\text{std.}$ ratios required to demonstrate noncompliance. As with consecutive full-period samples, Figure 3 shows there is a point of diminishing returns in attempting to reduce error in the mean by taking more than seven or so grab samples. However, since the level of variability in the mean of grab samples is usually much higher than for the same number of full-period samples, one might have to take many more than seven grab samples to attain the same level of precision afforded by even fewer than 4 to 7 full-period samples. Thus, we have a statistical criterion which can lead to economies in sampling by permitting reduction in sampling effort with a calculable degree of confidence. We can conclude that the optimum number of grab samples to be taken over the time period appropriate to the standard is 4 to 7 samples.

Finally, Figure 5 demonstrates the Partial Period Consecutive Samples Procedure. The effect of sample size and total time covered by all samples on requirements for demonstration of noncompliance is shown by the family of four curves. A typical sampling/analytical CV of 0.10 is used for all curves. The bottom curve (8-hour total sample time) is the same curve as the CV = 0.10 curve of Figure 4. The taking of partial period consecutive samples is a compromise between the preferred full-period sample(s) and the least desirable grab samples. If one assumes a GSD of 2.5 on Figure 3, we see that a curve of about 5 1/2 hours on Figure 5 would have approximately the same \bar{x}/std ratios. Therefore if one cannot sample for at least 70% of the time period appropriate to the standard (5 1/2 hours for an 8-hour standard) it is better to go to a grab sampling strategy.

Sampling periods

The last question to be answered concerns when to take the grab samples during the period of exposure. The accuracy of the probability level for the test depends upon implied assumptions of the log-normality and independence of the sample results which are averaged. These assumptions are not highly restrictive if precautions are taken to avoid bias when selecting the sampling times over the period for which the standard is defined. To this end, it is desirable to choose the sampling periods in a statistically random fashion. For a standard which is defined as a time-weighted average concentration over a period longer than the sampling interval, an unbiased estimate of the true average can be assured by taking samples at random intervals. It is valid to sample at equal intervals if the series is known to be stationary, i.e., if contaminant levels vary randomly about a constant mean and fluctuations are of short duration relative to length of the sampling interval. However, if means and their confidence limits were to be calculated from samples taken at equally spaced intervals, biased results could occur if cycles in the operation were in phase with the sampling periods. Results from random sampling are valid even when cycles and trends occur during the period of the standard. The word random refers to the manner of selecting the sample. Any particular sample could be the outcome of a random sampling procedure. A practical way of defining random sampling is that any portion of the work shift has the same chance of being sampled as any other.

What do we do if we can't sample during the entire work day or entire length of the operation? Strictly speaking, the measurement results are valid only for the duration of the period when the measurements were taken. However, professional judgement may allow inferences to be made about concentrations during other portions of the day. Reliable knowledge concerning the operation is required to make these types of extrapolations.

To select a true random sample, proceed as follows:

1) Divide the total period over which the standard is defined into (n) mutually exclusive (non-overlapping) intervals whose collective lengths equal the period for the standard. The number (n) is equal to (P/s) , where P is the period of the standard and s is the length of sampling intervals.

For example, if 15-minute samples are taken and the standard is a time-weighted average over an 8-hour work period, there would be $n = 32$ possible sampling intervals from which a random sample could be selected.

2) Number the possible sampling intervals consecutively: 1,2,3, . . . , n. For example, for an 8-hour standard over a workday from 8:00 am to 4:30 pm with 12:00 N to 12:30 pm spent outside the work area for lunch, we would assign the following code numbers for 15-minutes sampling intervals.

<u>Code #</u>	<u>Interval</u>
1	8:00 - 8:15 am
2	8:15 - 8:30 am
3	8:30 - 8:45 am
.	
.	
15	11:30 - 11:45 am
16	11:45 - 12:00 N
17	12:30 - 12:45 pm
18	12:45 - 1:00 pm
.	
.	
31	4:00 - 4:15 pm
32	4:15 - 4:30 pm

3) If (n) random samples are to be taken, use a table of random numbers such as Table A-36 in Natrella (28). Select a "subjectively random" starting point and from there list the first (n) different integers between 1 and (n).

For example, suppose we want to select five random 15-minute sampling periods from 32 possible periods. Arbitrarily choose the first column and the eleventh row (67) from the first page of Natrella's (28) Table A-36 as our starting point. This page is reproduced in this report as Figure 6. Moving vertically downward in the table our five periods would be 24, 6, 29, 16, and 4.

<u>Period</u>	<u>Time Interval</u>
4	8:45 - 9:00 am
6	9:15 - 9:30 am
16	11:45 - 12:00 N
24	2:15 - 2:30 pm
29	3:30 - 3:45 pm

Juda and Budzinski (19) give a similar procedure.

CEILING STANDARDS

Samples taken for determination of noncompliance with ceiling standards are treated in a manner similar to those taken for comparison with time-weighted average (TWA) standards. Two important differences should be noted.

The first point is that samples taken for comparison with ceiling standards are best taken in a nonrandom fashion. That is, all available knowledge relating to the area, individual, and process being sampled should be utilized to obtain samples during periods of maximum expected concentrations of the substance.

The second point is that samples taken for comparison to ceiling standards are normally taken for a much shorter time period than those taken for calculating time-weighted averages (TWA). There are three different ways in which the time period for a ceiling standard may be defined (29 CFR 1910.93) (see 29).

29 CFR 1910.93 (a)(1) for Table G-1:

No time period. "An employee's exposure . . . shall at no time exceed the ceiling value. . ."

29 CFR 1910.93 (b)(2) for Table G-2:

No time period, but peak above the "ceiling" allowed. "An employee's exposure . . . shall not exceed at any time during an 8-hour shift the acceptable ceiling concentration limit . . . except for . . . a maximum peak value.

29 CFR 1910.93 (b)(2) for Table G-2:

Short time period (5 to 30 minutes) defined as "maximum duration" for "maximum peak". The ceiling standard directly above may be exceeded for short periods up to a concentration defined as "acceptable maximum peak above the acceptable ceiling concentration for an 8-hour shift".

However under the current Joint NIOSH/OSHA Standards Completion Program all ceiling standard substances in Table G-1 of 29 CFR 1910.93 will have the standard defined for 15 minute time periods as:

"concentrations not in excess of . . . averaged over any 15-minute period during an 8-hour work shift."

Measurements taken for the purpose of determining employee exposure to ceiling standard substances should be taken during periods of maximum expected airborne concentrations of the substance. Each measurement should consist of a 15-minute sample (or series of consecutive samples totaling 15 minutes) taken in the employee's breathing zone (air that would most nearly represent that inhaled by the employee). A minimum of 3 measurements should be taken on one work shift and the highest of all measurements taken is a good estimate of the employee's upper exposure for that shift.

Taking 3 measurements on a shift makes it easier to spot gross errors or mistakes. However, in most cases only the highest value would be statistically tested for noncompliance by the Full-Period Sample Procedure. If the samples are taken for comparison to the "maximum peak" ceiling standard the sampling period should equal the "maximum duration" period for that particular standard. Thus, in the case of detector tubes it might be necessary to take several consecutive samples and average the results. Then the Full-Period Consecutive Samples Procedure would be used to analyze the results.

TREATMENT OF GAS DETECTOR TUBE DATA

Because gas detector tubes receive quite widespread usage in the industrial hygiene field and because their accuracy and precision have been subject of much controversy, it was decided to treat them separately. Under existing Federal Regulations (42 CFR Part 84) (see 30), stringent requirements are set forth for the construction and performance of gas detector tube units. One of the most important requirements pertains to the accuracy of the units. 42 CFR 84.20 (e) states:

"Accuracy of gas detector tubes shall be such that measurements made by these tubes, used in accordance with the applicant's instructions, are reliable to within plus or minus 25% of the actual value at concentrations of 1, 2, and 5 times the test standard of the contaminant of interest, and within plus or minus 35% of the actual value at one-half the test standard."

However, it is important to note that the above are maximum allowable error specifications and that the majority of tubes meeting all the construction and performance requirements of the proposed regulations will do considerable better than plus or minus 25% accuracy.

Johnson and Roper (31, 32, 33, 34) at the National Institute for Occupational Safety and Health have conducted performance studies to determine the reliability of hydrogen sulfide, chlorine, ozone, and trichloroethylene gas detector tubes. Data was taken from the above referenced four reports for the specific brands of gas detector tubes that met existing federal regulations and was subjected to an analysis of variance statistical procedure. The data involved in the analysis was for 31 "batches" (minimum of ten tubes per batch) from five manufacturers for four different gases at three concentration levels.

The analysis of the detector tube data showed that, after assuming variability is constant where concentrations are expressed as percentages of the true value, the total variability can be resolved into two components of variance:

- 1) Systematic batch errors (interbatch variation) with variance $(s^2)_{\text{sys}}$
- 2) Random (tube-to-tube) errors and other analytical errors (intrabatch variation) with variance $(s^2)_{\text{tube}}$.

The interbatch variation was found to have a mean for all batches relatively close to zero (95% confidence limits: 0.7% to 6.9%) as compared to the range of systematic errors from batch to batch. The interbatch variation had a standard deviation $s_{sys} = 7.2\%$. The intrabatch variation (tube-to-tube) had a standard deviation $s_{tube} = 14.0\%$.

However, the variance of actual environmental data will include both the variation due to environmental fluctuations and the random intrabatch (tube-to-tube) variations. Thus, the only variation that should be "added" to the data variation is the interbatch standard deviation of 7.2%. But after the appropriate equations were derived and utilized, it was found that the standard deviation of actual data usually is much larger (by factors of 10 or 20) than the 7.2%. It was found by ignoring the 7.2% in the calculations, the maximum error introduced was less than 5%. Thus, we feel that the data for detector tubes can be adequately and confidently analyzed by the Grab Samples Procedure in Appendix A with no additional modifications. When the Full-Period Sample or Full-Period Consecutive Samples Procedure is used a coefficient of variation of 14% (from Table 2) should be used.

Procedures for Data Analysis

A single sample or the time-weighted average of several consecutive samples taken for the entire time period for which a standard is defined yields the "best" estimate of the true average concentration of the airborne contaminant. This type of sample is referred to as a "full-period" sample. Typically, a full-period sample would have to be 8 to 36% above the standard in order to demonstrate noncompliance with 95% confidence. The Full-Period Single Sample Procedure is used to calculate the 95% lower confidence limit (LCL) for this type of sample. If several consecutive samples are taken for the entire time period of a standard the Full-Period Consecutive Samples Procedure is used to calculate the 95% Lower Confidence Limit (LCL) for the arithmetic mean of the samples. The greater the number of consecutive samples, the less the mean of those samples has to be above the standard in order to demonstrate noncompliance. Thus, it is better to take two consecutive 4-hour samples than one 8-hour and better yet to take four consecutive 2-hour samples during an 8-hour period for which a standard is defined. This is true only if the costs of sample analysis are negligible or not considered.

In some cases it isn't possible to take the sample(s) for the entire period of the standard. If one samples for six to almost eight hours for an 8-hour standard, the sample(s) is referred to as a "partial period" sample(s). The sample (s) is analyzed in the same manner as a full-period sample(s), but the LCL is compared to a "partial period limit" as calculated by the Partial Period Consecutive Sample(s) Procedure.

Finally, if long term (greater than one hour) samples are not practical one must estimate the average air concentration by several short term or grab samples. A suitable number of grab samples is at least four to seven. The Grab Samples Procedure is used to calculate the LCL for arithmetic mean of these samples. Samples taken for comparison to a TWA (time-weighted average) standard should be taken at random intervals over the period of the standard. However, samples taken for comparison with a ceiling standard should be taken during periods of maximum expected concentrations.

Equations for the various procedures are given in sections immediately following. Refer to Figure 7 when deciding what data analysis procedure should be used.

Full-Period Single Sample Procedure

The following procedure should be used to determine noncompliance with either a TWA, ceiling, or excursion standard. It is used when only one sample is being tested. For a TWA standard the sample must have been taken for the entire period for which the standard is defined (usually 8 hours). The variability (standard deviation or coefficient of variation C.V.) of the sampling and analytical methods used to collect and analyze the sample must be well known from previous measurements. The statistical test given is one-sided comparison-of means test using the normal distribution at the 95% confidence level.

ONLY IF THE LCL OF THE SAMPLE EXCEEDS THE STANDARD ARE WE 95% CONFIDENT THAT THE TRUE AVERAGE CONCENTRATION EXCEEDS THE STANDARD AND THAT A CONDITION OF NONCOMPLIANCE EXISTS.

$$LCL = x - 1.645\sigma$$

where 1.645 = critical standard normal deviate for 95% confidence,

σ = standard deviation of sampling/analytical method
which is well known from prior data, and

x = measurement being tested.

If the coefficient of variation (C.V.) is known the LCL is computed from:

$$LCL = x - ((1.645)(C.V.)(standard))$$

Figure 8 can be used to aid this calculation (n=1). Some coefficients of variation are available from Table 2.

If the sample is taken for less than the entire period for which the standard is defined, refer to the Partial Period Sample Procedure which is a variation of the Full-Period Consecutive Samples Procedure.

Table 2. Coefficients of Variation for Some Sampling/Analytical Procedures.

<u>Sampling/Analytical</u>	<u>C.V.</u>	<u>Data Source</u>
Colorimetric detector tubes	0.14	A
Rotameter on personal pumps	0.05	B
Charcoal tubes (sampling/analytical)	0.10	C
Asbestos (sampling/counting)	0.22	D
Respirable dust except coal mine dust (sampling/weighing)	0.09	E
Gross dust (sampling/weighing)	0.05	E

Data Source References

- A. Analysis of data from (31,32,33, and 34) as discussed previously in this report.
- B. NIOSH Engineering Branch estimate of typical calibrated pumps capable of the range 1.5 to 3.0 lpm.
- C. Conservative estimate by the authors. Recent work under NIOSH Contract CDC-99-74-75 have shown typical CVs (precision only) of 0.055 to 0.09 for charcoal tubes.
- D. Reference (25).
- E. NIOSH Engineering Branch estimate based on the use of pumps in the flow range 1.5 to 3.0 lpm and a collected mass of a least 1.0 milligram.

Full-Period Consecutive Samples Procedure

The following procedure should be used to determine noncompliance with either a TWA, ceiling or excursion standard. It is used when several consecutive samples are taken for the entire time period for which the standard is defined. If the samples do not cover the entire time period of the standard, refer to the Partial Period Samples Procedure in the next section. The variability (standard deviation or coefficient of variation) of the sampling and analytical methods used to collect and analyze the samples must be well known from previous measurements. The statistical test given is a one-sided comparison-of-means test using the normal distribution at the 95% confidence level.

ONLY IF THE LCL OF THE MEAN OF THE CONSECUTIVE SAMPLES EXCEEDS THE STANDARD ARE WE 95% CONFIDENT THAT THE TRUE AVERAGE CONCENTRATION EXCEEDS THE STANDARD AND THAT A CONDITION OF NONCOMPLIANCE EXISTS.

$$LCL = \bar{x} - 1.645\sigma_{\bar{x}}$$

where T_i = duration of i th sample

x_i = measurement of concentration in i th sample

$T = T_1 + T_2 + \dots + T_n$ = total of durations for the n consecutive samples

\bar{x} = time weighted average of n samples

$$= (1/T) (T_1x_1 + T_2x_2 + \dots + T_nx_n)$$

σ = standard deviation of sampling/analytical method which is well known from prior data

$$\sigma_{\bar{x}} = (\sigma/T)(T_1^2 + T_2^2 + \dots + T_n^2)^{\frac{1}{2}}$$

$$= \sigma/(n)^{\frac{1}{2}} \text{ if } T_1 = T_2 = \dots = T_n$$

If the coefficient of variation (C.V.) is known, σ is computed from:

$$\sigma = (\text{C.V.}) (\text{standard})$$

Figure 8 can be used to aid this calculation in the case of n equal-duration samples. Some coefficients of variation are available from Table 2.

Partial- Period Consecutive Sample(s) Procedure

One or a series of consecutive samples collected over less than the period for which a standard is defined is referred to as a "partial period" sample(s). Since it is known with certainty that the concentration during the period not covered by the sample could not be less than zero, the 8 hour average standard is multiplied by the following factor to obtain a conservative partial period standard. The LCL is then calculated as in the previous section and compared to the partial period standard.

$$\text{Factor} = \frac{\text{(time period of the standard)}}{\text{(actual time of the sample(s))}}$$

For an 8-hour standard typical factors would be:

<u>Total time of sample(s)</u>	<u>Factor</u>
8.00 hours	1.00
7.75	1.032
7.50	1.067
7.25	1.103
7.00	1.143
6.75	1.185
6.50	1.231
6.25	1.280
6.00	1.333

See Example #2 for the use of the above technique.

Grab Sample Data Analysis

As stated previously, if full period samples of industrial contaminant concentrations are available, the best method of modeling the uncertainties of the result is with a normal distribution. The appropriate statistical testing procedures were presented earlier in this report. However, when only a set of grab samples is available, the lognormal distribution best describes the uncertainties of the process. Draft versions of this report contained a procedure to compute a conservative lower confidence limit (LCL) on the average contaminant level from a number of grab samples. The method was conservative in that the confidence limits were rather wide since exact statistical theory was not available. However during the long developmental period of this report the results of a NIOSH contract became available. Partial results of this contract are presented as Appendix A so that all data analysis basic procedures will be available to the reader in one report.

Appendix A presents a Grab Samples procedure for estimating the average concentration of a contaminant and making a decision on the level of the contaminant. The theory underlying this procedure was developed by Systems Control, Inc. and reported in full in (21, 22). The main advantages of this procedure are:

1. It is contaminant-independent. Thus, it becomes possible to use only a single decision chart for every contaminant.
2. It is capable of both the Non-compliance Decision and the No Action Decision. Each of these decisions is subject to a predetermined probability of Type I or Type II error. A Type I error is said to occur if the non-compliance decision is wrongly asserted. A Type II error is said to occur if the no action decision is wrongly asserted.
3. The estimation and decision procedures are implemented via a simple and straightforward nomographic method. For estimation, the procedure yields the best estimate of the actual average contaminant level.

A minor disadvantage of the procedure is that the LCL is not directly computed as in previous procedures. However it is felt the simplicity of the calculations required and the plotting of a single point on the decision chart far outweighs any advantages the direct calculation of an LCL would yield. Although the authors normally dislike a "black box" approach we feel most users of these procedures prefer simplicity and speed in reading a decision concerning sample results.

Example #1 for Consecutive Full-Period Samples Procedure

Hazardous concentrations of benzene were suspected in a plant. It was decided to use the personal charcoal tube sampling method with the low volume pump so that long-term (4-hour) samples could be taken. Several pumps were used and 4-hour samples were taken both in the morning and afternoon.

<u>Employee</u>	<u>Morning</u>	<u>Afternoon</u>	<u>TWA</u>
A	12 ppm	14 ppm	13 ppm benzene
B	21	25	23
C	25	21	23
D	10	12	11

The 1974 8-hour TWA standard for benzene is 10 ppm (29 CFR 1910.93).

First step

From Table 2 we obtain the C.V. for the charcoal tube method as 0.10. To obtain σ we multiply the standard (10 ppm) by 0.10 for a σ of 1.0 ppm. First test the two highest TWA's of 23 ppm with the Consecutive Full-Period Samples Procedure.

Second step

The LCL on the mean of 23 ppm is,

$$\begin{aligned} \text{LCL} &= 23 \text{ ppm} - (1.645) (1.0 \text{ ppm}) / (2)^{\frac{1}{2}} \\ &= 23 - 1.2 = 21.8 \text{ ppm} \end{aligned}$$

This could also have been done by reading the 1.2 ppm from Fig 8. For employee A the LCL is (13 - 1.2) and for employee D the LCL is (11 - 1.2 = 9.8).

Third step

For three of the four employees (A,B,C) the LCL on the means was higher than the 8-hour TWA Standard. We can conclude with 95% confidence that the true 8-hour TWA did exceed the standard for these employees and that a condition of noncompliance did exist.

Example #2 for Full-Period (Ceiling) Samples (Part a) and Partial Period Consecutive Samples Procedures (Part b)

The following data was obtained on one employee at one operation in an asbestos plant:

<u>Sample No. (i)</u>	<u>Time Period</u>	<u>Duration T_i (min.)</u>	<u>Conc X_i (f/cc)</u>
1	0848 - 0925	37	17.5
2	0925 - 0952	27	15.5
3	0957 - 1039	42	8.0
4	1039 - 1105	26	17.5
5	1105 - 1137	32	19.0
6	1140 - 1149	9	32.0
7	1320 - 1341	21	10.0
8	1341 - 1425	44	6.8

The 1972 standards for asbestos are 1) 5 f/cc for 8-hour TWA and 2) 10 f/cc ceiling never to be exceeded. The samples represent a total sample time of 238 minutes out of a 440 minute exposure day (40 minutes for lunch).

Part a Ceiling Limit (Full Period Samples)

Note that OSHA recommends that samples taken for comparison to the ceiling standard be taken for a minimum of 15 minutes. Sample #6, taken over only 9 minutes, is nevertheless included in this example for illustrative purposes.

First Step

First analyze all the samples except #3, #7, and #8 for violation of the ceiling standard by the Full-Period Procedure. Calculate the σ by multiplying the C.V. of 0.22 (Table 2) by the standard of 10 f/cc. We obtain $\sigma = 2.2$ f/cc. From Fig. 8 we see we need to subtract 3.6 f/cc from each sample to obtain the LCL for that sample. Alternatively, calculate $(1.645)(2.2) = 3.62$ or 3.6.

Second step

For the five samples:

<u>Sample No.</u>	<u>Conc (f/cc)</u>	<u>LCL (f/cc)</u>
1	17.5	13.9
2	15.5	11.9
4	17.5	13.9
5	19.0	15.4
6	32.0	28.4

Third step

For all five samples the LCL is greater than the ceiling standard of 10 f/cc and we are 95% confident that the standard has been violated in each case.

Part b 8-Hour Average Limit (Partial Period Consecutive Samples)

Samples 1 through 8 have a TWA concentration of 15.8 f/cc. This is the mean of 8 measurements that span an entire 238 minute time period.

First step

Calculate the partial period limit for the 238 minute period:

$$\text{Limit} = (5 \text{ f/cc}) (480/238) = 10.1 \text{ f/cc}$$

Second step

Calculate the LCL on the mean of the 8 consecutive measurements:

$$\bar{x} = (3262.2/238) = 13.7 \text{ f/cc}$$

$$\sigma = (\text{C.V.})(\text{limit}) = (0.22) (10.1) = 2.2$$

$$\sigma_{\bar{x}} = (2.2/238)(8,020)^{\frac{1}{2}} = 0.83$$

$$\begin{aligned} \text{LCL} &= \bar{x} - 1.645\sigma_{\bar{x}} \\ &= 13.7 - (1.645)(0.83) \\ &= 13.7 - 1.4 = 12.3 \end{aligned}$$

Third step

The LCL (12.3 f/cc) on the mean of the 8 samples (13.7 f/cc) is greater than the limit for the 238 minute period (10.1 f/cc). Thus the employer is in violation of the 8-hour average standard and a citation should be issued.

Example #3 for Full-Period Consecutive Samples Procedure

Colorimetric detector tubes were used to sample for hydrogen sulfide. The standards for hydrogen sulfide are 1) 50 ppm for a maximum duration of 10 minutes (once only if no other measurable exposure occurs) and 2) 20 ppm acceptable ceiling concentration for an 8-hour shift. Three (3) detector tubes were used consecutively during each of three ten minute sampling periods during periods of expected peak exposure. The following results were obtained:

<u>Period</u>	<u>Samples</u>	<u>10 Min. Mean Conc.</u>
0905 - 0915 hrs.	5,10,5 ppm	6.7 ppm
1110 - 1120	35,55,15	35.0
1320 - 1330	20,25,30	25.0

First step

Since the samples were obtained in a nonrandom fashion they should not be used for the determination of noncompliance with an 8-hour standard (this is academic since there was no 8-hour standard for hydrogen sulfide). From Table 2 we obtain the coefficient of variation (C.V.) for colorimetric detector tubes as 0.14.

Second step

Test the three values taken during the second period for noncompliance with the acceptable ceiling concentration standard of 20 ppm with the Full-Period Consecutive Samples Procedure. The mean of the three values is 35 ppm. The σ is found by multiplying the C.V. of 0.14 by the 20 ppm standard. We obtain a σ of 2.8 ppm.

The LCL is:

$$\begin{aligned} \text{LCL} &= 35 \text{ ppm} - (1.645)(2.8 \text{ ppm}) / (3)^{\frac{1}{2}} \\ &= 32.3 \text{ ppm.} \end{aligned}$$

(Alternatively, 2.7 ppm could have been read from Figure 8 and subtracted from 35 ppm.)

Fourth step

Since the LCL of the mean value of the three tubes (32.2 ppm) exceeds the standard (20 ppm), we can conclude with 95% confidence that the true average concentration during the 10-minute period did exceed the ceiling standard. A citation could be issued because a condition of noncompliance exists.

REFERENCES

1. Roach, S.A.: "Testing Compliance with the ACGIH Threshold Limit Values for Respirable Dusts Evaluated by Count," Trans. ACGIH, 27-39(1966).
2. Roach, S.A.: "A More Rational Basis for Air Sampling Programs," AIHA J., 27, 1-12 (1966).
3. Roach, S.A., Baier, E.J., Ayer, H.E., and R.L. Harris: "Testing Compliance with Threshold Limit Values for Respirable Dusts", AIHA J, 28, 543-550 (1967).
4. Kerr, G.W.: "Use of Statistical Methodology in Environmental Monitoring", AIHA J, 23, 75-82 (1962).
5. Larsen, R.I.: "A Method for Determining Source Reduction Required to Meet Quality Standards", JAPCA, 11, 71 (1961).
6. Larsen, R.I.: " A New Mathematical Model of Air Pollutant Concentration Averaging Time and Frequency", JAPCA, 19, 24 (1969).
7. Phinney, D.E. and J.E. Newman: "The Precision Associated with the Sampling Frequencies of Total Particulate at Indianapolis, Indiana", JAPCA, 22, 692-695 (1972).
8. Larsen, R.I.: "A Mathematical Model for Relating Air Quality Measurements to Air Quality Standards", AP-89, Environmental Protection Agency, U.S. Gov't Printing Office (1971).
9. Breslin, A.J., Ong, L., Glauberman, H., George, A.C., and P. LeClare: "The Accuracy of Dust Exposure Estimates Obtained from Conventional Air Sampling", AIHA J, 28, 56-61 (1967).
10. Sherwood, R.J.: "On the Interpretation of Air Sampling for Radioactive Particles", AIHA J, 27, 98-109 (1966).
11. Sherwood, R.J.: "The Monitoring of Benzene Exposure by Air Sampling". AIHA J, 32, 840-846 (1971).
12. Jones, A.R. and R.S. Brief: "Evaluating Benzene Exposures", AIHA J, 32, 610-613, (1971).
13. Gale, H.J.: "The Lognormal Distribution and Some Examples of Its Application in the Field of Radiation Protection", Atomic Energy Research Establishment Report AERE-R 4736, Her Majesty's Stationery Office (1965).

14. Gale, H.J.: "Some Examples of the Application of the Lognormal Distribution in Radiation Protection", Ann. Occup. Hyg., 10, 39-45 (1967)
15. Coenen, W.: "The Confidence Limits for the Mean Values of Dust Concentration", Staub, 26, 39-45 (Engl. Translation) (May 1966).
16. Coenen, W.: "Measurement Assessment of the Concentration of Health-Impairing, Especially Silicogenic Dusts at Work Places of Surface Industries", Staub, 31, 16-23 (Engl. Translation) (December 1971).
17. Hounam, R.F.: "An Application of the Log-Normal Distribution to Some Air Sampling Results and Recommendations on the Interpretation of Air Sampling Data", Atomic Energy Research Establishment Report AERE-M 1469, Her Majesty's Stationary Office (1965).
18. Juda, J. and K. Budzinski: "Fehler bei der Bestimmung der mittleren Staubkonzentration als Funktion der Anzahl der Einzelmessungen" (Errors in Determining the Mean Dust Concentration as a Function of the Number of Individual Measurements), Staub, 24, 283-287, (August 1964).
19. Juda, J. and K. Budzinski: "Determining the Tolerance Range of the Mean Value of Dust Concentration", Staub, 27, 12-16 (Engl. Translation) (April 1967).
20. LeClare, P.L., Breslin, A.J., and L. Ong: "Factors Affecting the Accuracy of Average Dust Concentration Measurements", AIHA J, 30, 386-393 (1969).
21. Bar-Shalom, Y., Segall, A., Budenaers, D., and R.B. Shainker: "Statistical Theory for Sampling of Time-Varying Industrial Atmospheric Contaminant Levels," Systems Control, Inc., Final Report for NIOSH Contract HSM 99-73-78, Palo Alto, CA, June 1974.
22. Budenaers, D., Bar-Shalom, Y., Segall, A., and R.B. Schainker: "Handbook for Decisions on Industrial Atmospheric Contaminant Exposure Levels," Systems Control, Inc., Final Field Handbook for NIOSH Contract HSM 99-73-78, Palo Alto, CA, June 1974.
23. Leidel, N.A.: "Statistics in Compliance", presented at the 1974 American Industrial Hygiene Conference, Miami Beach, Florida, 14 May 74.
24. Leidel, N.A.: "Optimum Sampling Times for Airborne Asbestos Fibers", USPHS, NIOSH TR-82 (1973).

25. Leidel, N.A., Bayer, S. G., and R.D. Zumwalde: "USPHS/NIOSH Membrane Filter Method for Evaluating Airborne Asbestos Fibers", USPHS, NIOSH Tr-84 (1973).
26. Larsen, R.I., C.E. Zimmer, D.A. Lynn, and K. G. Blemel: "Analyzing Air Pollutant Concentration and Dosage Data", JAPCA, 17, 85-93 (1967).
27. Saltzman, B.E.: "Significance of Sampling Time in Air Monitoring", JAPCA, 20, 660-665 (1970).
28. Natrella, M.G.: "Experimental Statistics", NBS Handbook #91, U.S. Gov't Printing Office (1963).
29. Federal Register, 39, 23540-23583 (27 June 74).
30. "NIOSH Certified Personal Protective Equipment", NIOSH Publication 75-119, July 74 (with Sept. 74 and Jan. 75 Supplements).
31. Johnson, B.A.: The Evaluation of Gas Detector Tube Systems: Hydrogen Sulfide, USPHS/NIOSH RR-18 (1972).
32. Johnson, B.A., and Roper, P.: The Evaluation of Gas Detector Tube Systems: Chlorine, USPHS/NIOSH RR-19 (1972).
33. Johnson, B.A.: The Evaluation of Gas Detector Tube Systems: Ozone, USPHS/NIOSH RR-21 (1972).
34. Roper, P.: The Evaluation of Gas Detector Tube System: Trichloroethylene, USPHS/NIOSH RR-22 (1972).

Notes on availability of references.

- a) References 21 and 22 will be available from NIOSH as NIOSH Technical Report 75 - 147, a combined publication.
- b) References 24 and 25 are to be reprinted by NIOSH as a combined publication under the title listed for reference 25.

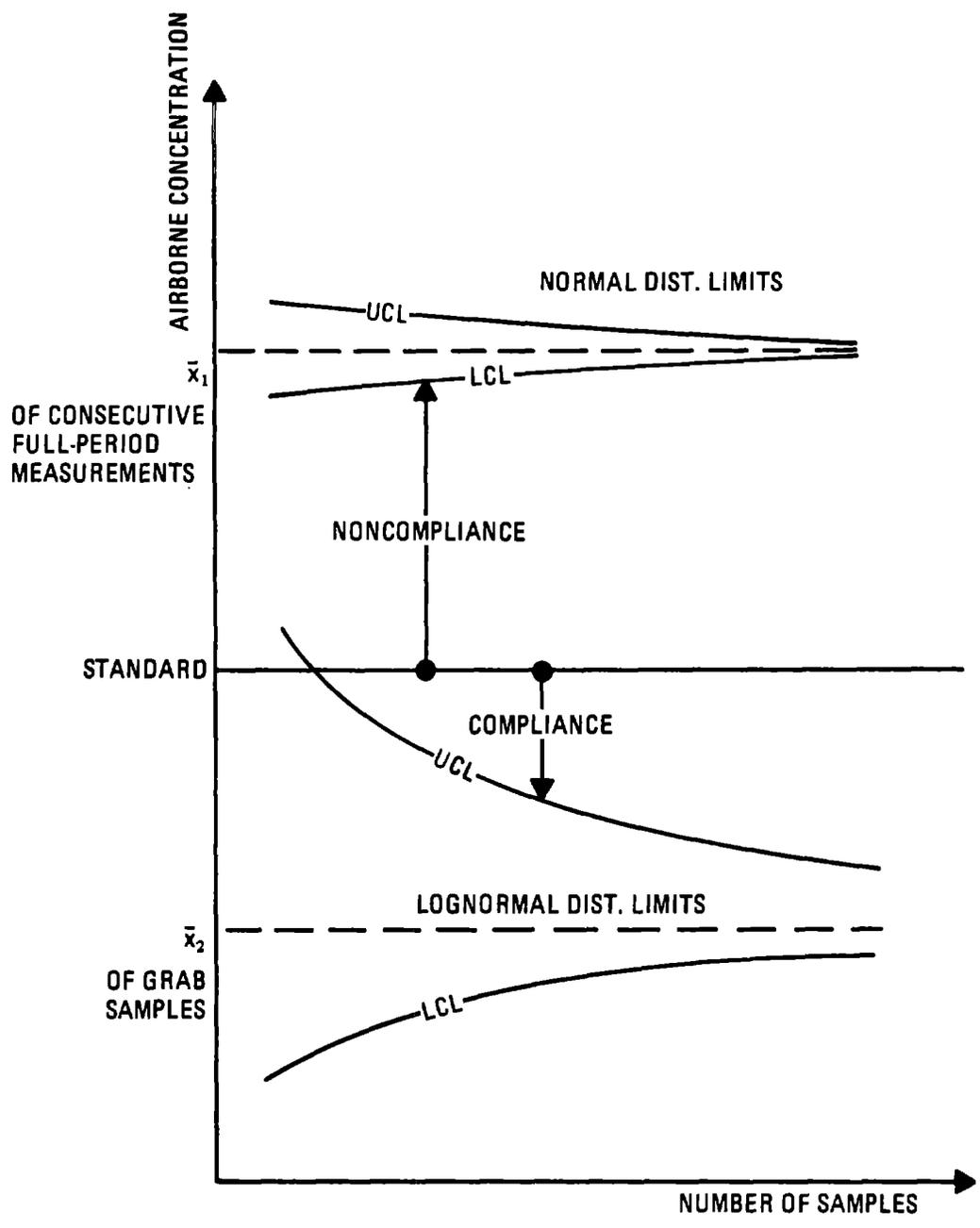


Figure 2 - Compliance and noncompliance situations for two types of sampling strategies.

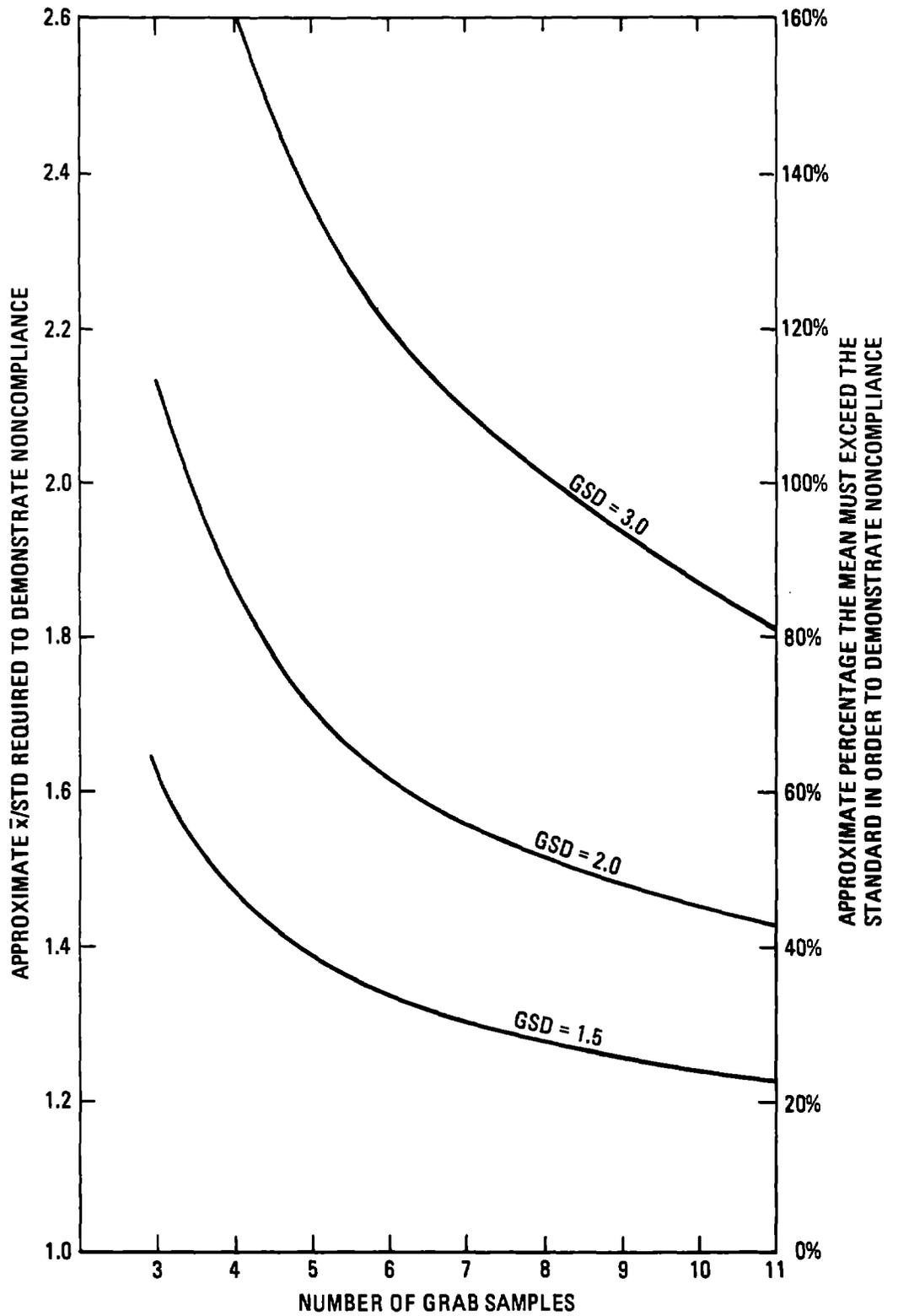


Figure 3 - Effect of small grab sample sizes on requirements for demonstration of noncompliance. Three different data geometric standard deviations (GSD) are shown which reflect the amount of variability in the environment.

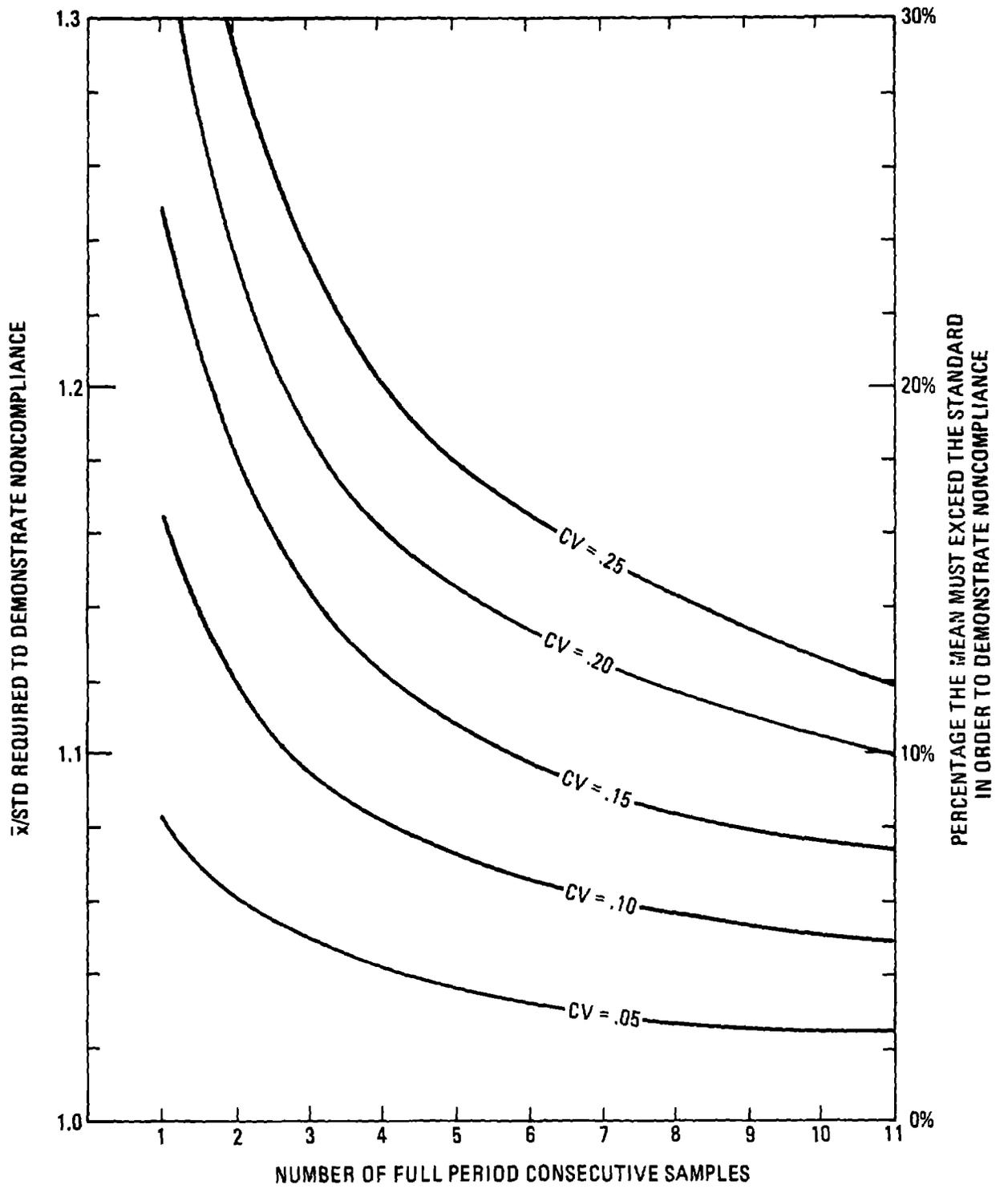


Figure 4 - Effect of Full Period Consecutive Sample size on requirements for demonstration of noncompliance.

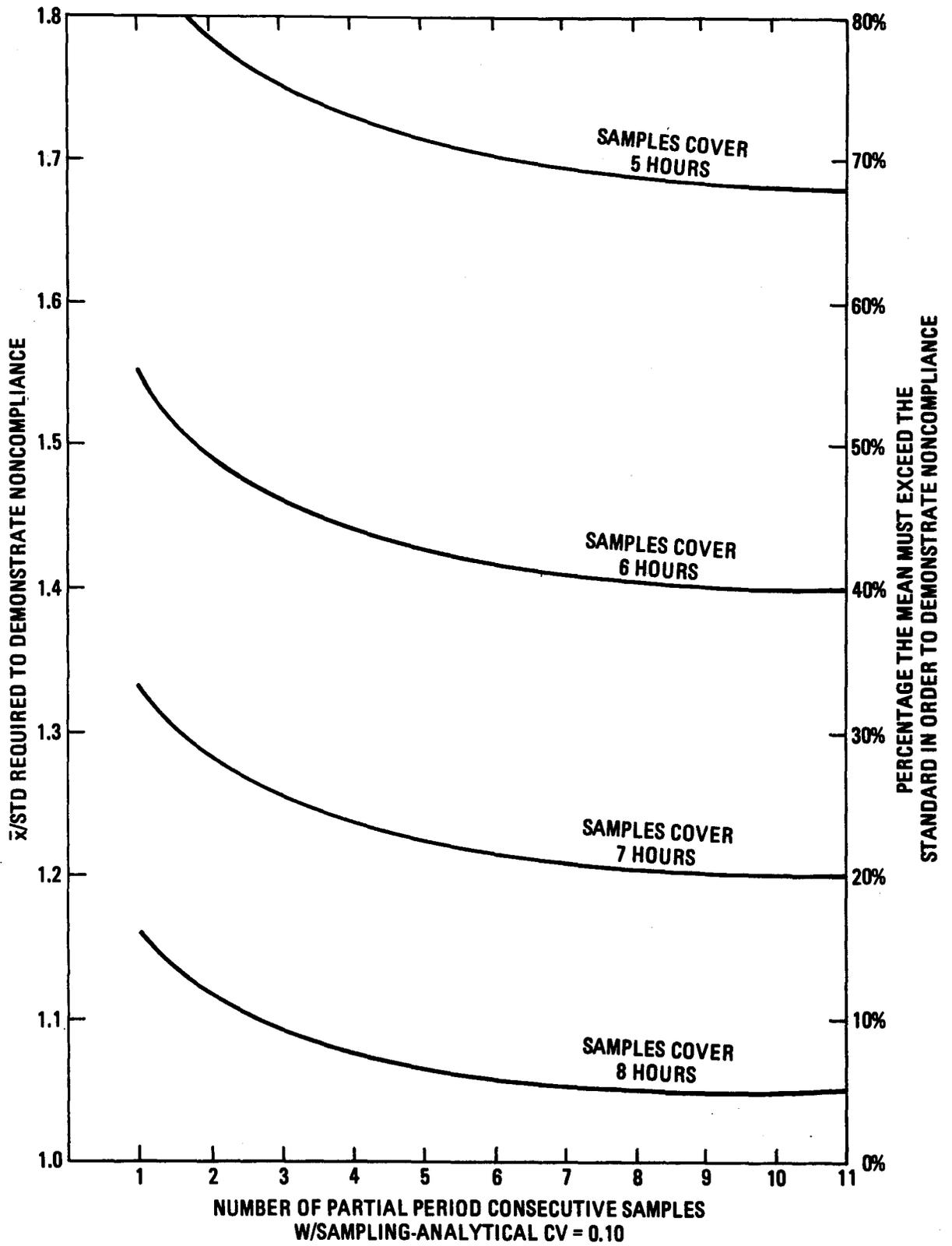


Figure 5 - Effect of Partial Period Consecutive Sample size and total time covered by all samples on requirements for demonstration of noncompliance.

TABLE A-36. SHORT TABLE OF RANDOM NUMBERS

46	96	85	77	27	92	86	26	45	21	89	91	71	42	64	64	58	22	75	81	74	91	48	46	18
44	19	15	32	63	55	87	77	33	29	45	00	31	34	84	05	72	90	44	27	78	22	07	62	17
34	39	80	62	24	33	81	67	28	11	34	79	26	35	34	23	09	94	00	80	55	31	63	27	91
74	97	80	30	65	07	71	30	01	84	47	45	89	70	74	13	04	90	51	27	61	34	63	87	44
22	14	61	60	86	38	33	71	13	33	72	08	16	13	50	56	48	51	29	48	30	93	45	66	29
40	03	96	40	03	47	24	60	09	21	21	18	00	05	86	52	85	40	73	73	57	68	36	33	91
52	33	76	44	56	15	47	75	78	73	78	19	87	06	98	47	48	02	62	03	42	05	32	55	02
37	59	20	40	93	17	82	24	19	90	80	87	32	74	59	84	24	49	79	17	23	75	83	42	00
11	02	55	57	48	84	74	36	22	67	19	20	15	92	53	37	13	75	54	89	56	73	23	39	07
10	33	79	26	34	54	71	33	89	74	68	48	23	17	49	18	81	05	52	85	70	05	73	11	17
67	59	28	25	47	89	11	65	65	20	42	23	96	41	64	20	30	89	87	64	37	93	36	96	35
93	50	75	20	09	18	54	34	68	02	54	87	23	05	43	36	98	29	97	93	87	08	30	92	98
24	43	23	72	80	64	34	27	23	46	15	36	10	63	21	59	69	76	02	62	31	62	47	60	34
39	91	63	18	38	27	10	78	88	84	42	32	00	97	92	00	04	94	50	05	75	82	70	80	35
74	62	19	67	54	18	28	92	33	69	98	96	74	35	72	11	68	25	08	95	31	79	11	79	54
91	03	35	60	81	16	61	97	25	14	78	21	22	05	25	47	26	37	80	39	19	06	41	02	00
42	57	66	76	72	91	03	63	48	46	44	01	33	53	62	28	30	59	55	05	02	16	13	17	54
06	36	63	06	15	03	72	38	01	58	25	37	66	48	56	19	56	41	29	28	76	49	74	39	50
92	70	96	70	89	80	87	14	25	49	25	94	62	78	26	15	41	39	48	75	64	69	61	06	38
91	08	88	53	52	13	04	82	23	00	26	36	47	44	04	08	84	80	07	44	76	51	52	41	59
68	85	97	74	47	53	90	05	90	84	87	48	25	01	11	05	45	11	43	15	60	40	31	84	59
59	54	13	09	13	80	42	29	63	03	24	64	12	43	26	10	01	55	62	07	79	83	05	59	61
39	18	32	69	33	46	58	19	34	03	59	28	97	31	02	65	47	47	70	39	74	17	30	22	65
67	43	31	09	12	60	19	57	63	78	11	80	10	97	15	70	04	89	81	78	54	84	87	83	42
61	75	37	19	56	90	75	39	03	56	49	92	72	95	27	52	87	47	12	52	54	62	43	23	13
78	10	91	11	00	63	19	63	74	58	69	03	51	38	60	36	53	56	77	06	69	03	89	91	24
93	23	71	58	09	78	08	03	07	71	79	32	25	19	61	04	40	33	12	06	78	91	97	88	95
37	55	48	82	63	89	92	59	14	72	19	17	22	51	90	20	03	64	96	60	48	01	95	44	84
62	13	11	71	17	23	29	25	13	85	33	35	07	69	25	68	57	92	57	11	84	44	01	33	66
29	89	97	47	03	13	20	86	22	45	59	98	64	53	89	64	94	81	55	87	73	81	58	46	42
16	94	85	82	89	07	17	30	29	89	89	80	98	36	25	36	53	02	49	14	34	03	52	09	20
04	93	10	59	75	12	98	84	60	93	68	16	87	60	11	50	46	56	58	45	88	72	50	46	11
95	71	43	68	97	18	85	17	13	08	00	50	77	50	46	92	45	26	97	21	48	22	23	08	32
86	05	39	14	35	48	68	18	36	57	09	62	40	28	87	08	74	79	91	08	27	12	43	32	03
59	30	60	10	41	31	00	69	63	77	01	89	94	60	19	02	70	88	72	33	38	88	20	60	86
05	45	35	40	54	03	98	96	76	27	77	84	80	08	64	60	44	34	54	24	85	20	85	77	32
71	85	17	74	66	27	85	19	55	56	51	36	48	92	32	44	40	47	10	38	22	52	42	29	96
80	20	32	80	98	00	40	92	57	51	52	83	14	55	31	99	73	23	40	07	64	54	44	99	21
13	50	78	02	73	39	66	82	01	28	67	51	75	66	33	97	47	58	42	44	88	09	28	58	06
67	92	65	41	45	36	77	96	46	21	14	39	56	36	70	15	74	43	62	69	82	30	77	28	77
72	56	73	44	26	04	62	81	15	35	79	26	99	57	23	22	25	94	80	62	95	48	98	23	86
28	86	85	64	94	11	58	78	45	36	34	45	91	38	51	10	68	36	87	81	16	77	30	19	36
69	57	40	80	44	94	60	82	94	93	98	01	48	50	57	69	60	77	69	60	74	22	05	77	17
71	20	03	30	79	25	74	17	78	34	54	45	04	77	42	59	75	78	64	99	37	03	18	03	36
89	98	55	98	22	45	12	49	82	71	57	33	28	69	50	59	15	09	25	79	39	42	84	18	70
58	74	82	81	14	02	01	05	77	94	65	57	70	39	42	45	56	84	31	59	18	70	41	74	60
50	54	73	81	91	07	81	26	25	45	49	61	22	88	41	20	00	15	59	93	51	60	65	65	63
49	33	72	90	10	20	65	28	44	63	95	86	75	78	69	24	41	65	86	10	34	10	32	00	93
11	85	01	43	65	02	85	69	56	38	34	29	64	35	48	15	70	11	77	53	01	34	82	91	04
34	22	46	41	84	74	27	02	57	77	47	93	72	02	95	63	75	74	69	69	61	34	31	92	13

Adapted with permission from *A Million Random Digits* by The Rand Corporation, Copyright, 1955, The Free Press.

Fig. 6 - First page of Table A-36 in Natrella (28).

**DATA ANALYSIS PROCEDURE
TO BE USED**

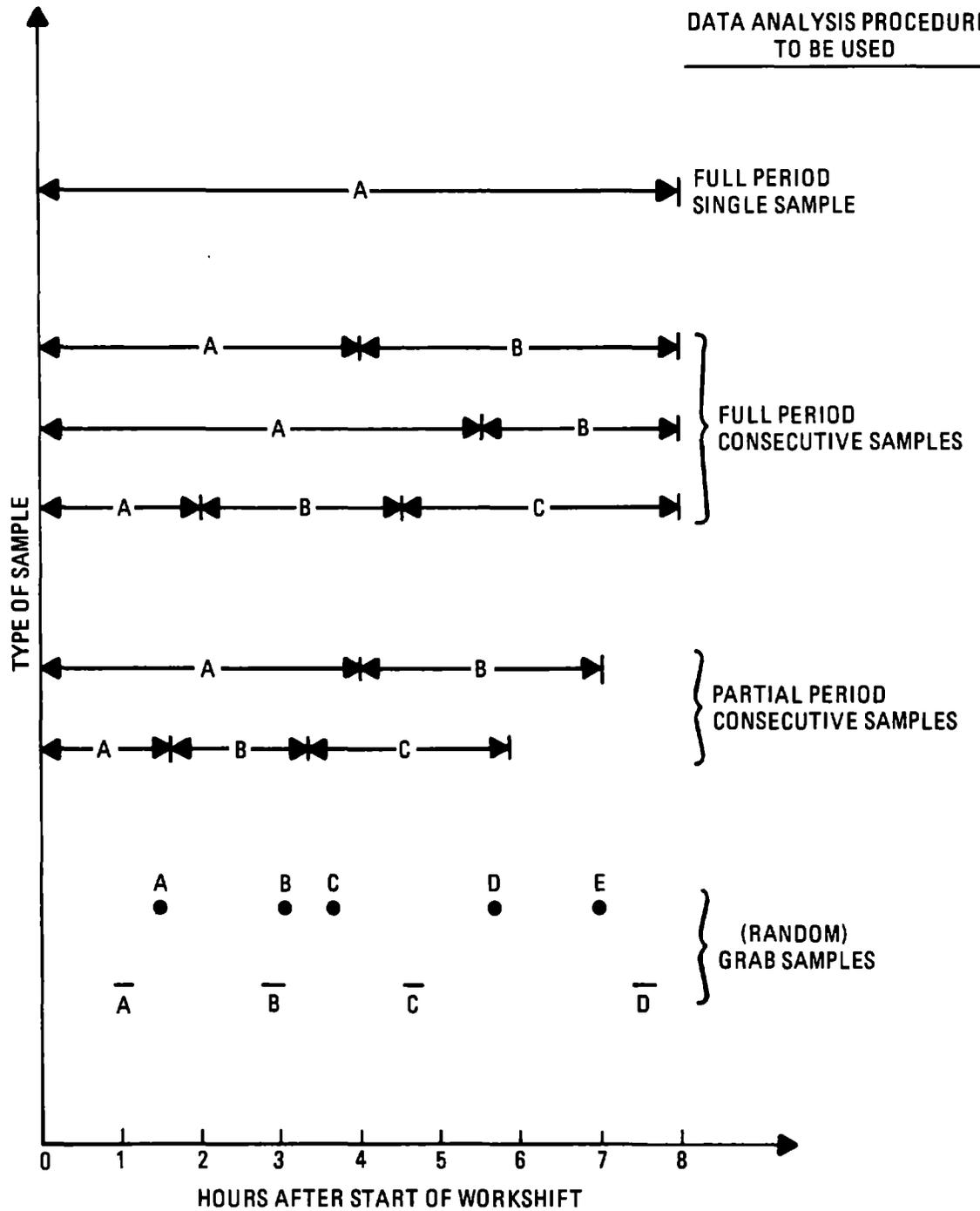


Figure 7 - Reference chart of data analysis procedures to use for an 8-hour average exposure standard.

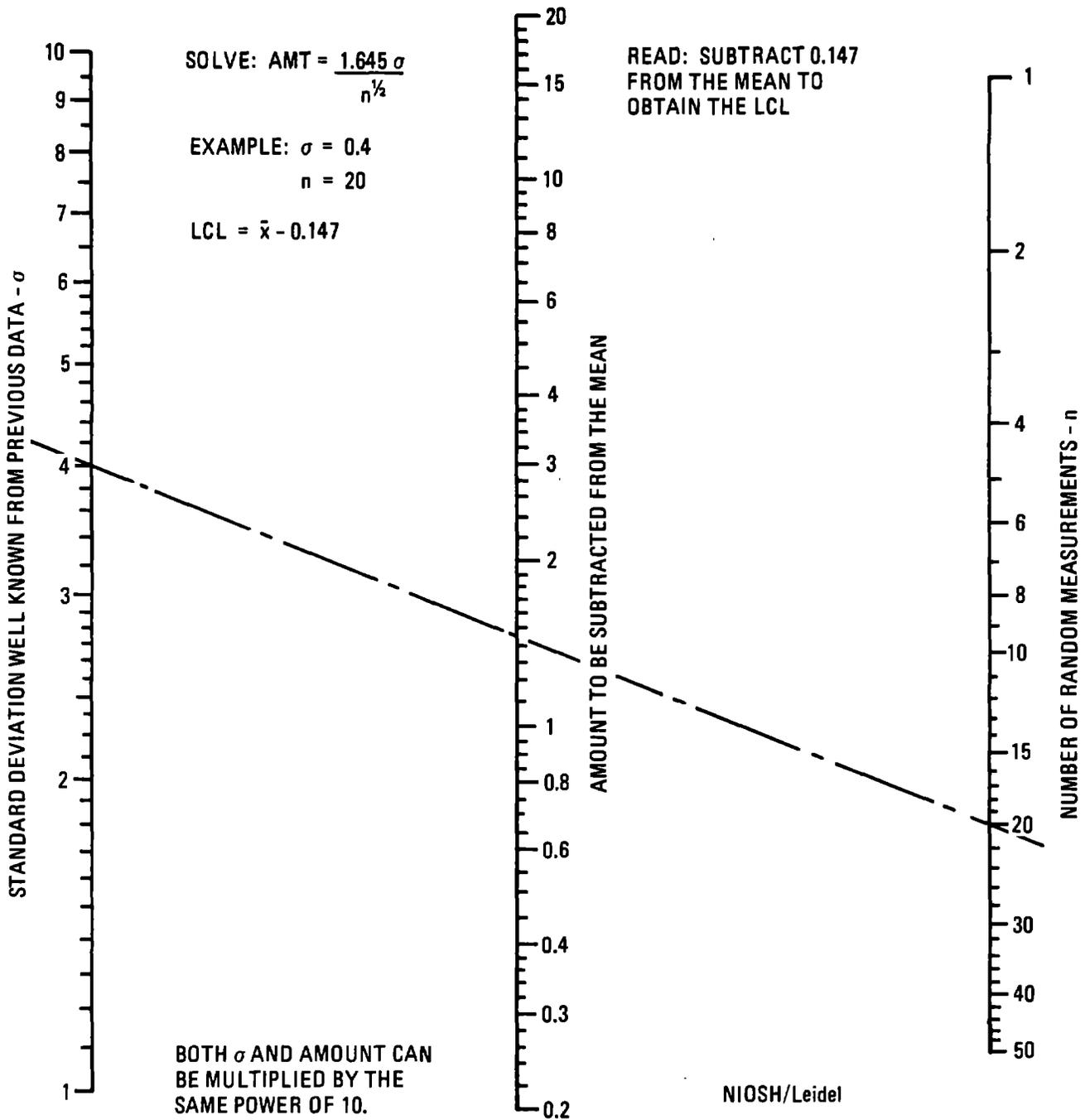


Figure 8 - Nomogram for Full Period or Partial Period procedures using a well known standard deviation.

APPENDIX A - GRAB SAMPLES PROCEDURE

0.0 DEFINITIONS

This section contains a Definition of Terms and Notations used in this Appendix. Section 0.1 contains the Definition of Terms. The term or phrase being defined is underlined. Section 0.2 contains a definition of all mathematical notations used in this Appendix.

0.1 DEFINITION OF TERMS

Average concentration: Arithmetic mean of concentration over an 8-hour period.

Federal Standard (for 8-hour exposures): The highest level of average concentration to which a worker can be exposed (29 CFR 1910.93).

Relative Concentration: Measured concentration divided by the Federal Standard.

A non-compliance Decision: A decision made with a given maximum low probability of error that a worker's contaminant exposure level exceeds the Federal Standard.

A No Action Decision: A decision made with a given maximum low probability of error that a worker's contaminant exposure does not exceed the Federal Standard.

No Decision: A decision that neither compliance nor non-compliance can be asserted with sufficiently low probabilities of making the respective incorrect decisions.

Stationary (time-invariant) Data: A set of data for which the mean and variance do not change with time.

Normally Distributed Variable: A variable which is said to be normally distributed has fluctuations which have a bell-shaped frequency distribution which is characterized fully by the mean and standard deviation.

Lognormally Distributed Variable: A variable is said to be log-normally distributed if the logarithm of the variable is normally distributed.

0.2 NOTATIONS

STD: Federal Standard such as 29 CFR 1910.93

M : Average concentration

\hat{M} : Best estimate of the average concentration

n : Number of concentration samples

s : Sample standard deviation

t_i : Midpoints of sampling periods during which the concentrations X_i are obtained

X_i : Measurements of the concentrations

x_i : Ratio between measurements and the Federal Standard

y_i : Logarithm (base 10) of x_i

\bar{y} : Sample Mean of y_i

1. INTRODUCTION

This Appendix presents an easy to follow procedure for making decisions regarding the average of a time varying industrial contaminant exposure level. By using this Appendix it will be possible to make statistical decisions as to whether or not a particular contaminant level exceeds the exposure standards set forth by OSHA regulations 29 CFR 1910.93. A list of definitions of terms and concepts used in this handbook is presented in Section 0. The procedures in this Appendix will allow one to make one of the following three decisions regarding a worker's contaminant exposure level:

- . Non-compliance
- . No Action
- . No Decision

By "Non-compliance" it is meant that the decision has been reached at a given high confidence level that the worker's contaminant exposure level exceeds the Federal Standard. By "No Action" it is meant that the decision has been reached with a given high confidence level that the worker's contaminant exposure level does not exceed the standard. Finally, by the "No Decision" choice it is meant that neither of the above choices can be asserted with a sufficiently high confidence level.

In addition to the three-way decision procedure described above, this handbook will provide a nomographic method of determining the best estimate of the actual average level of contaminant.

The decision and estimation procedures described in this handbook cover the case where the contaminant data is a set of grab samples from a time-varying industrial environment. A lognormal distribution is assumed for the variability of individual grab samples taken at random intervals.

The step-by-step procedures to be followed in making decisions regarding contaminant exposure level and in obtaining estimates of average contaminant level are described in Section 2. Examples of procedure are provided in Section 3.

The suggested method of using this procedure is as follows:

Review Section 2 to gain an overview of the methodology without undue concern about understanding all the procedural details.

Then, work through the examples in Section 3, to obtain insight into the methodology presented in Section 3.

If necessary, review Section 2 to solidify the procedure.

2. DESCRIPTION OF THE PROCEDURE

This section describes the procedure for estimating and making decisions on the average concentration of an industrial contaminant. An outline of the procedure is presented in Section 2.1. A detailed description of each of the steps in the procedure is presented in Section 2.2. A collection of examples on the use of the procedure is presented in Section 3.

2.1 OUTLINE OF PROCEDURE

The procedure is divided into the following six steps. The interrelationship between the steps is illustrated in the flow diagram in Figure 1.

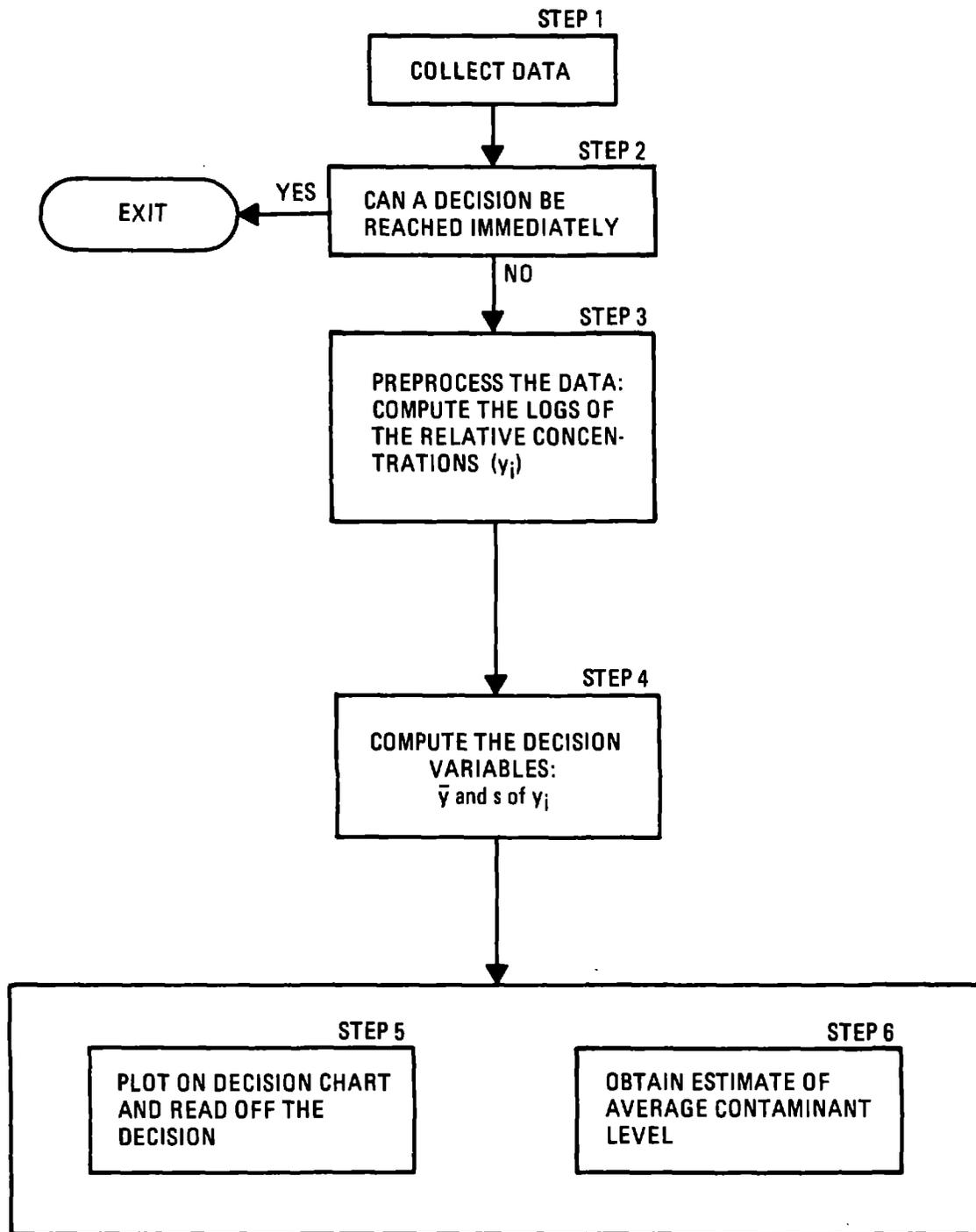
- Step 1: Collect data, then proceed to Step 2.
- Step 2: Check whether a decision can be reached immediately; if not, proceed to Step 3.
- Step 3: Preprocess the data by dividing the concentration data by the Federal Standard and computing the common logarithms of these relative concentrations, then proceed to Step 4.
- Step 4: Compute the decision variables, go to Step 5.
- Step 5: Make the decision by plotting a single point on the decision chart and read off the decision (either non-compliance, no action, or no decision).
- Step 6: Obtain the estimate of the average contaminant level from the estimation nomograph.

2.2 DETAILED DESCRIPTION OF PROCEDURE

Step 1 - Collect Data

The available contaminant data consists of the following information:

- (a) The concentration measurements: X_1, \dots, X_n (average concentrations for the sampling periods).
- (b) The times at which these measurements were taken: t_1, \dots, t_n (the midpoints of the sampling periods).



Appendix A Figure 1 - Flow chart for Grab Samples Procedure.

Technical Remark: One should not attempt to reach a decision about the eight-hour average contaminant level based upon measurements that cover only a small portion (e.g., last two hours) of the eight hour work day.

Step 2 - Can a Decision Be Reached Immediately?

In some cases it is possible to quickly "eyeball" the concentration data to determine that a state of noncompliance exists. If the minimum of the concentration data is 30% above the standard, then a state of non-compliance probably exists and an exit from the rest of the steps can usually be made.

Step 3 - Preprocess the Data

In this step the data is preprocessed to prepare for later analysis.

- (a) Calculate the relative concentrations with respect to the applicable Federal Standard.

Let the Federal Standard for the contaminant being investigated be denoted by STD. Then compute the following quantities:

$$x_1 = \frac{X_1}{STD}, \quad x_2 = \frac{X_2}{STD}, \quad \dots \quad x_n = \frac{X_n}{STD}$$

That is, divide each of the concentration measurements by the standard. The new x_1, x_2, \dots, x_n are called the relative concentrations. The reason that this division is performed is to make the concentrations of contaminant independent of the particular contaminant being investigated. The "standard" value for the "new" x_1, x_2, \dots, x_n will always be one.

- (b) Compute the common logarithms of the relative concentrations.

In this part of Step 3, the common logarithms of the relative concentrations are determined. The logarithms of the relative concentrations are denoted by y_1, y_2, \dots, y_n . Therefore,

$$y_1 = \log x_1, \quad y_2 = \log x_2, \quad \dots \quad y_n = \log x_n$$

Step 4 - Compute the Decision Variables

Obtain the sample mean, denoted by \bar{y} , and the sample standard deviation, denoted by s . \bar{y}, s , and n are the decision variables.

These variables will be used in determining which of the following decisions should be made (Step 5).

- . non-compliance
- . no action
- . no decision

Also, these variables will be used to estimate the average concentration of contaminant (Step 6).

The computational formula for \bar{y} is:

$$\bar{y} = \frac{1}{n} (y_1 + y_2 + \dots + y_n) \quad (3)$$

The computational formula for s is:

$$s = \sqrt{\frac{1}{n-1} [(y_1 - \bar{y})^2 + (y_2 - \bar{y})^2 + \dots + (y_n - \bar{y})^2]} \quad (4a)$$

Or, in a computationally simpler form s is:

$$s = \sqrt{\frac{1}{n-1} [(y_1^2 + y_2^2 + \dots + y_n^2) - n(\bar{y})^2]} \quad (4b)$$

Step 5 - Make the Decision by Plotting the Decision Variables \bar{y} , s on the Decision Chart

Make the decision by plotting the decision variables \bar{y} , s on the decision chart, contained in Section 4.

The decision chart contains the following:

- . A vertical axis for the \bar{y} decision variable
- . A horizontal axis for the s decision variable
- . A set of curves which form the boundaries to the noncompliance, no decision, and no action decision regions. Each of these boundaries is a function of the number of observations. The number of observations is denoted by n . The values of n are from $n = 3$ to 25.
- . The decision region plot is set up to have the following probability structure:
 - (i) The probability of asserting "non-compliance", when true situation is "compliance", is at most 5%.
 - (ii) The probability of asserting "no action" when the true situation is "non-compliance" is at most 5%.

Technical Remark: Recall that (i) and (ii) are denoted technically as the size of Type I and Type II errors, respectively.

To use the decision region plot proceed as follows:

- . Plot the decision variables \bar{y} , s on the vertical and horizontal axis, respectively, to obtain the decision point.
- . If the decision point lies above the upper curve corresponding to the number of measurements n , then decide "non-compliance".
- . If the decision point lies below the lower curve corresponding to the number of measurements n , then decide "no action".
- . If the decision point is between two curves, then no decision can be made.
- . If the value of \bar{y} is out of range, either below $-.9$ or above $.3$, then usually "eyeball" interpolation will suffice to obtain the decision.
- . If the value of s is greater than $.5$ this indicates that one or more of the concentration measurements is "way out of line". This indicates usually that a state of non-compliance exists. Again, in this case, eyeball interpolation should be used.

Step 6 - Obtain Estimate of the Average Contaminant Level From The Estimation Nomograph.

The decision variables \bar{y} and s are also used to obtain the estimate of the value of the average contaminant level. The estimate is obtained using the estimation nomographs which are contained in Section 4. Estimation nomographs contain the following:

- . A left-hand vertical axis for the \bar{y} decision variable.
- . A horizontal axis for the s decision variable.
- . A right-hand vertical axis for reading off the average contaminant level denoted by \bar{M}/STD .
- . Each estimation nomograph has a value n associated with it, where n is the number of data points

The procedure for using an estimation nomograph is as follows:

- . Select the appropriate nomograph corresponding to the number of samples of the contaminant.

- . Plot the decision variables \bar{y} and s , with \bar{y} on the vertical axis and s on the horizontal axis.
- . Follow the nomograph curve nearest to the plotted point to the \hat{M}/STD axis on the right-hand side of the nomograph.
- . Interpolate between two values of \hat{M}/STD to obtain the appropriate \hat{M}/STD . If the value of \hat{M}/STD is multiplied by STD then the estimate of the average concentration M will be obtained.

If the values of \bar{y} or s are outside the range of the estimation nomograph, then the arithmetic mean of the measurements

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + \dots + x_n)$$

is to be used to estimate the average concentration.

3. EXAMPLES OF PROCEDURES

This section contains detailed examples of the procedure. Specific examples are provided to illustrate and demonstrate the use of the procedure for:

- . Computing the decision variables
- . Making the decision (i.e., either non-compliance, no action or no decision)
- . Obtaining estimates of contaminant level.

This section will also demonstrate in a step-by-step manner the organization of computations that are required for using the procedure as outlined in Figure 1. In particular, this section will illustrate:

- . The use of a computational worksheet
- . The use of the decision chart and estimation nomographs.

The examples illustrated in this section are intended to give the reader some insight into the way the methodology works.

3.1 Example 1

The measurements of carbon monoxide concentration yielded the data displayed in Table 3.1 for a particular employee in an industrial plant.

TIME OF DAY	9	10	11	12	13	14	15	16	17	18
CONCENTRATION OF CO (PPM)	45	50	75	40		85		20		

TABLE 3.1 RAW DATA FOR EXAMPLE #1

Table 3.1 summarizes the data collection, Step 1. For this example, the Time row in the table indicates the actual clock time at which the concentration measurements were obtained (mid-point of sampling interval).

The Federal Standard for CO is 50 ppm. By "eyeballing" the data above 50 ppm it cannot be determined if a state of non-compliance exists, therefore Step 3 must be done next.

Table 3.2 illustrates a worksheet for executing Step 3 and Step 5 (preprocessing the data and computing the decision). Columns 1 and 2 of the worksheet are the raw data recopied. Column 3 is the concentration divided by the Federal Standard, rounded to two places to the right of the decimal point.

Column 4 contains the value of the common logs (base 10) of the values in Column 3.

Step 4 must now be computed. Again the worksheet Table 3.2 is used to compute the decision variables. This is accomplished by computing column 5, which is the square of column 4 and adding the values of column 4 and 5 and putting the respective values in the bottom margins denoted by "SUM". The values for this example are -0.14 and 0.26, respectively. Note that all calculations are rounded to two significant figures. The values of the decision variables can now be computed using the equations (3) and (4b). The calculations are illustrated on the worksheet below Table 3.2. From the worksheet, the values of the three decision variables can be seen to be:

$$\begin{aligned}n &= 6 \\ \bar{y} &= -0.02 \\ s &= .23\end{aligned}$$

Remark on Calculations:

The following rules should be followed in performing the calculations on the worksheet.

- . Rule 1. Compute and round the logarithms to two places to the right of decimal point.
- . Rule 2. Compute the squares on the worksheet to four places to the right of the decimal point. Round to two places to the right of the decimal point after forming sum.
- . Rule 3. Compute the expressions for s on the worksheet to four places to the right of the decimal point. Round to two places to the right of decimal point after forming square root.
- . Rule 4. All other calculations are performed and rounded to two places past the decimal point. However, it may be necessary to carry additional significant digits in the terms of a difference if the leading digits of both terms are identical (as 22.15 and 22.34).

Note that the calculations on the worksheet can easily be implemented on advanced pocket scientific calculators which have hardwired programs for logs, arithmetic mean, and standard deviation. The worksheet is primarily set up for a calculator that performs only the elementary operations.

Now Steps 5 and 6 can be executed. Namely, in Step 5 the decision of non-compliance, no decision or no action can be determined; and in Step 6, an estimate of the average contaminant level can be determined.

Using the decision region plot in Section 4, plot the value of s on the horizontal axis and the value of \bar{y} on the vertical axis. Figure 3.1 illustrates the use of the plot. The result of plotting the decision variables is denoted by "A" in Figure 3.1. It is observed that for this example the "No Decision" choice must be made. This is because the point "A" lies between the boundaries of the $n = 6$ curves in the no decision region. Observe that the $n = 6$ curve must be interpolated between the $n = 5$ and $n = 7$ curve. This means that neither the non-compliance nor the no action decision can be asserted with sufficiently high confidence.

To estimate the average contaminant level, the estimation nomograph with $n = 5$ or 6 (nomograph with values of $n = 5$ or $n = 6$) must be used because the number of concentration data is equal to 6. Plotting s along the horizontal axis and \bar{y} along the vertical axis yields the point "B" in Figure 3.2. Reading the value corresponding to the curve on which "B" lies the value of \hat{M}/STD is seen to be equal to roughly $3/7$ of the way from the $1.0 \hat{M}/STD$ to the $1.2 \hat{M}/STD$ values.

Thus the value of \hat{M}/STD to be used is:

$$3/7 (1.2 - 1.0) + 1.0 = 1.09$$

This is the simple interpolation method.

This is the estimate of the average contaminant level divided by the standard. Since the standard is 50 ppm for CO, then $(1.09)(50) = 54.5$ ppm is the estimate of the average contaminant level for this data. Thus, it can be concluded that even though the estimated average level of CO above the standard, a state of non-compliance cannot be asserted. The arithmetic average of the contaminant can be seen to be 52.50 ppm. It is important to note that this arithmetic average is different than the average value of the contaminant.

1	2	3	4	5
TIME OF DAY	CONC.	$\frac{\text{CONC.}}{\text{STD}}$	$\text{LOG} \left[\frac{\text{CONC.}}{\text{STD}} \right]$	$(\text{COL. 4})^2$
9	45	0.90	-0.05	0.0025
10	50	1.00	0.00	0.0000
11	75	1.50	0.18	0.0324
12	40	0.80	-0.10	0.0100
14	85	1.70	0.23	0.0529
16	20	0.40	-0.40	0.1600
SUM			-0.14	0.26

TABLE 3.2

Decision Variables

$n = \underline{6}$ data values

$$\bar{y} = \frac{\text{Sum Col. 4}}{n} = \underline{-0.02}$$

$$s = \sqrt{\frac{1}{n-1} \left[(\text{Sum Col. 5}) - \frac{1}{n} (\text{Sum Col. 4})^2 \right]} = \underline{.23}$$

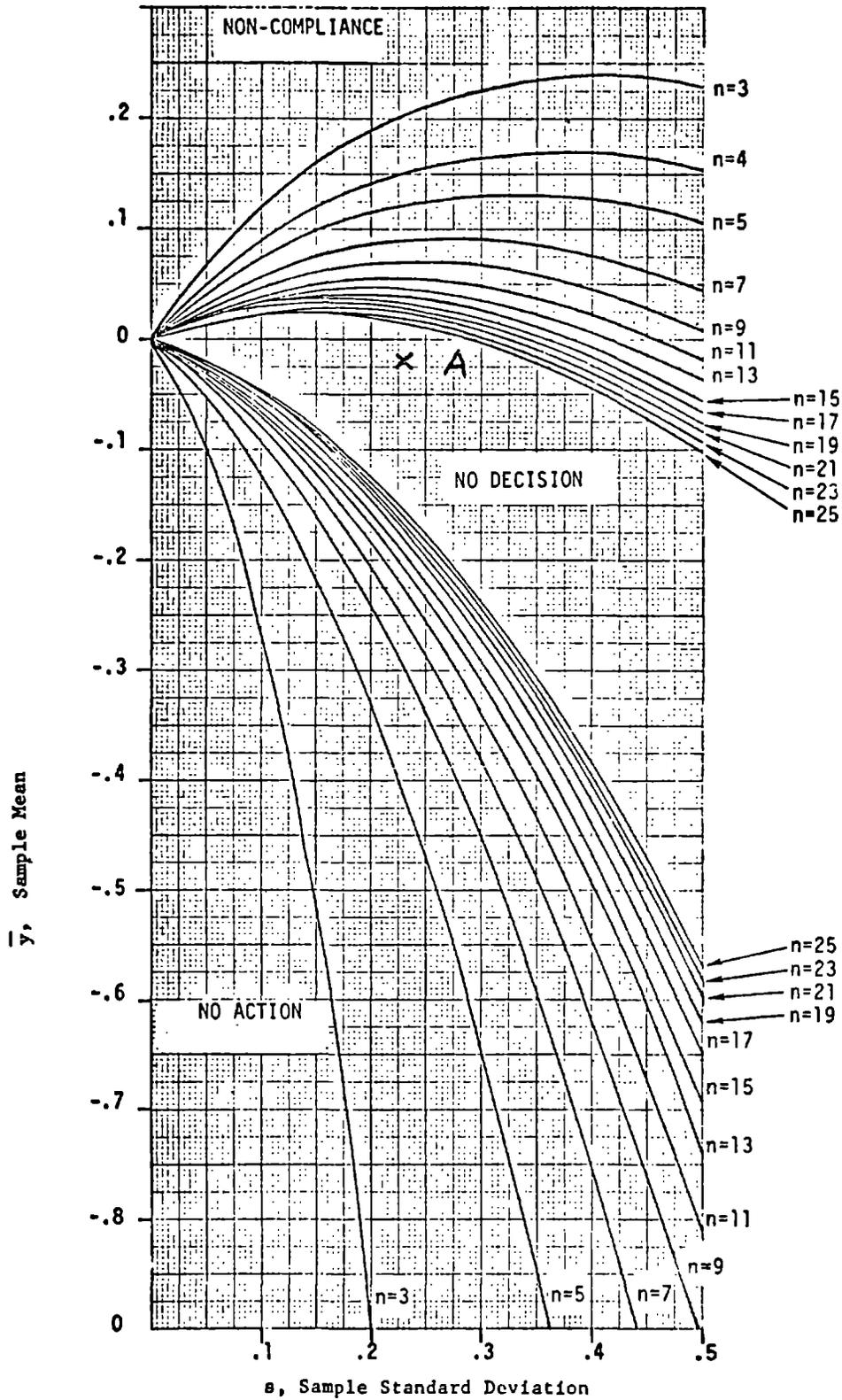


Figure 3.1 Decision regions

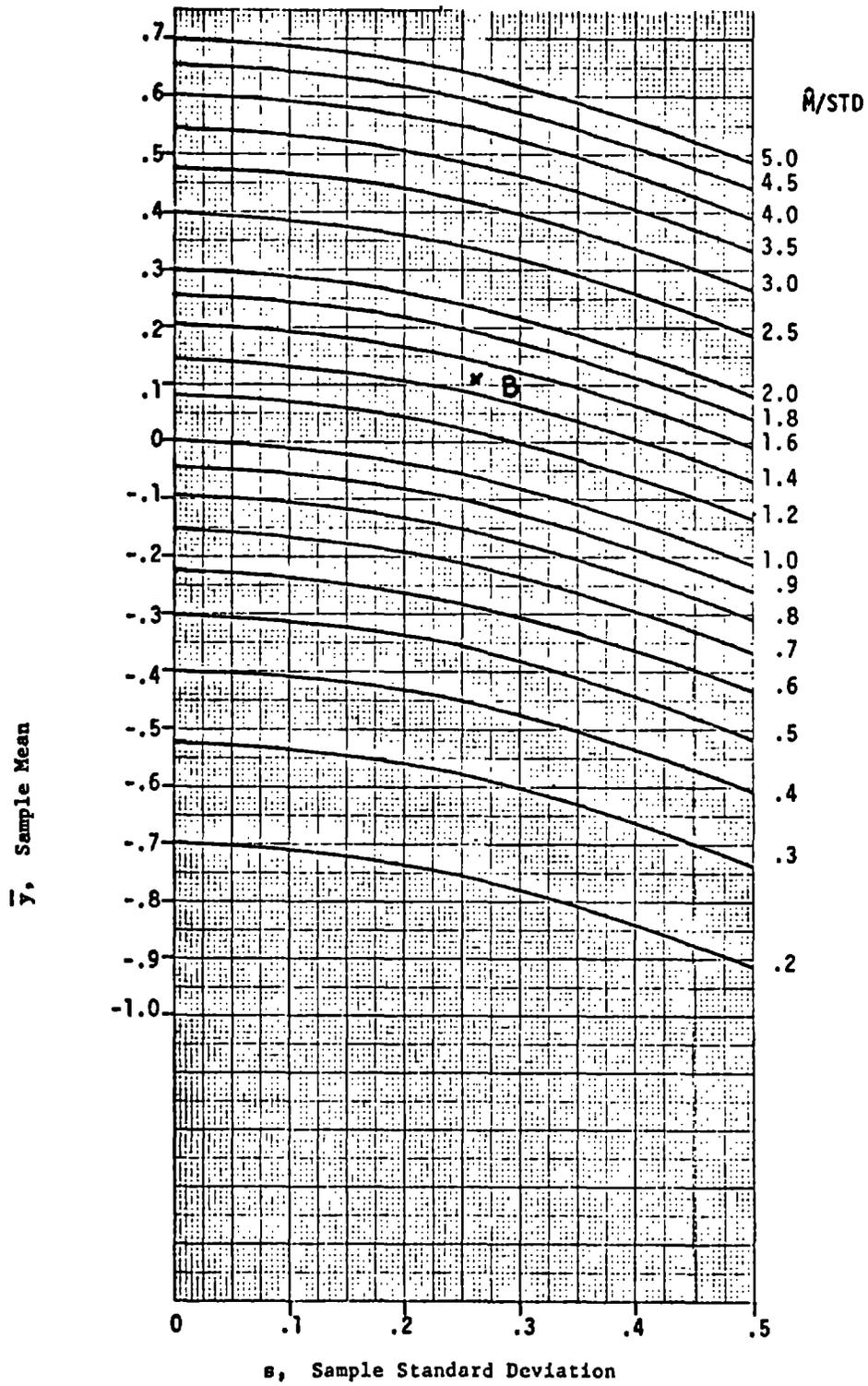


Figure 3.2 Estimation nomograph for Example #1 ($n = 6$)

Example #2

A Mercury vapor detector gave the following readings at the corresponding times in an industrial plant. The values are in mg/m³. The Federal Standard for Mercury vapor is 0.10 mg/m³.

TIME	9	10	11	12	13	14	15	16	17	18
CONCENTRATION OF Hg (mg/m ³)	.06	.20	.09			.16	.09	.30		

TABLE 3.3 RAW DATA FOR EXAMPLE #2

Since it cannot be decided if a state of non-compliance exists by merely looking at the data Steps 4,5, and 6 will be performed.

The worksheet is presented in Table 3.4. For detailed explanation of the worksheet see Example #1.

An examination of the decision chart, Figure 3.3, shows that for $n = 6, \bar{y} = .11$ and $s = .26$, a "non-compliance" decision must be asserted for this case. The decision point is plotted and denoted by "A" in Figure 3.3. This is because the point "A" lies above the interpolated curve for $n = 6$ on the decision region plot.

To estimate the average concentration of Hg vapor for this sample, the estimation nomograph with $n = 5$ or 6 , is to be used because the sample size is 6 . Using the estimation nomograph it can be seen that for $n = 6$, estimation point is plotted and denoted by "B" in Figure 3.4. Thus, using simple interpolation the value of the average concentration will be :

$$1.4 + .5 (1.6 - 1.4) = 1.50$$

The average value of the Hg concentration for the area sampled in mg/m³ is:

$$\hat{M} = \left(\frac{\hat{M}}{STD} \right) (STD) = (1.50)(0.10) = 0.15 \text{ mg/m}^3$$

rounded to two significant digits.

1	2	3	4	5
TIME OF DAY	CONC.	$\frac{\text{CONC.}}{\text{STD}}$	$\text{LOG} \left[\frac{\text{CONC.}}{\text{STD}} \right]$	$(\text{COL. 4})^2$
9	0.06	0.60	-0.22	0.0484
10	0.20	2.00	0.30	0.0900
11	0.09	0.90	-0.05	0.0025
14	0.16	1.60	0.20	0.0400
15	0.09	0.90	-0.05	0.0025
16	0.30	3.00	0.48	0.2304
SUM			.66	.41

TABLE 3.4

Decision Variables

$n = \underline{6}$ data values

$\bar{y} = \frac{\text{Sum Col. 4}}{n} = \underline{.11}$

$s = \sqrt{\frac{1}{n-1} \left[(\text{Sum Col. 5}) - \frac{1}{n} (\text{Sum Col. 4})^2 \right]} = \underline{.26}$

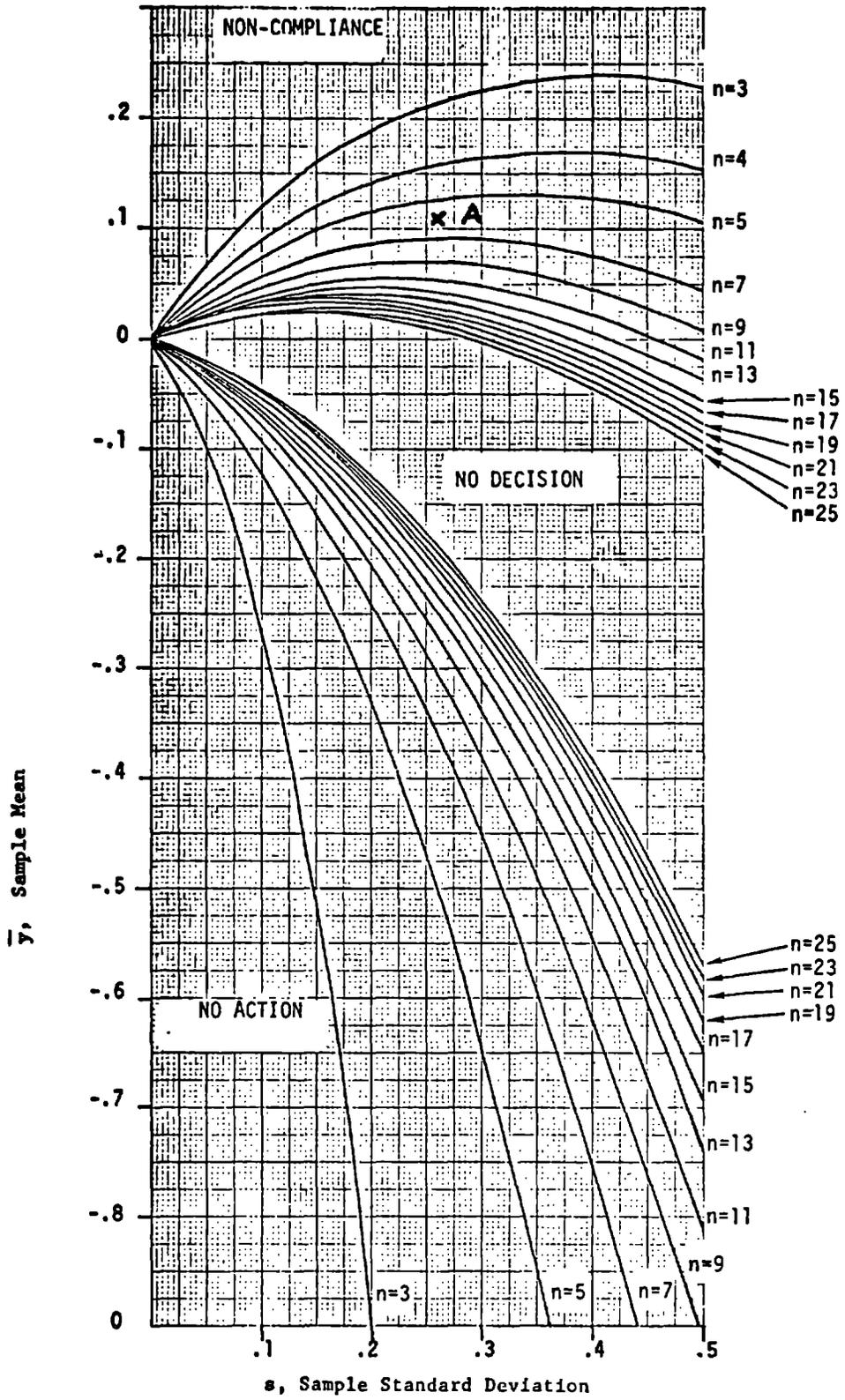


Figure 3.3 Decision regions for Example #2

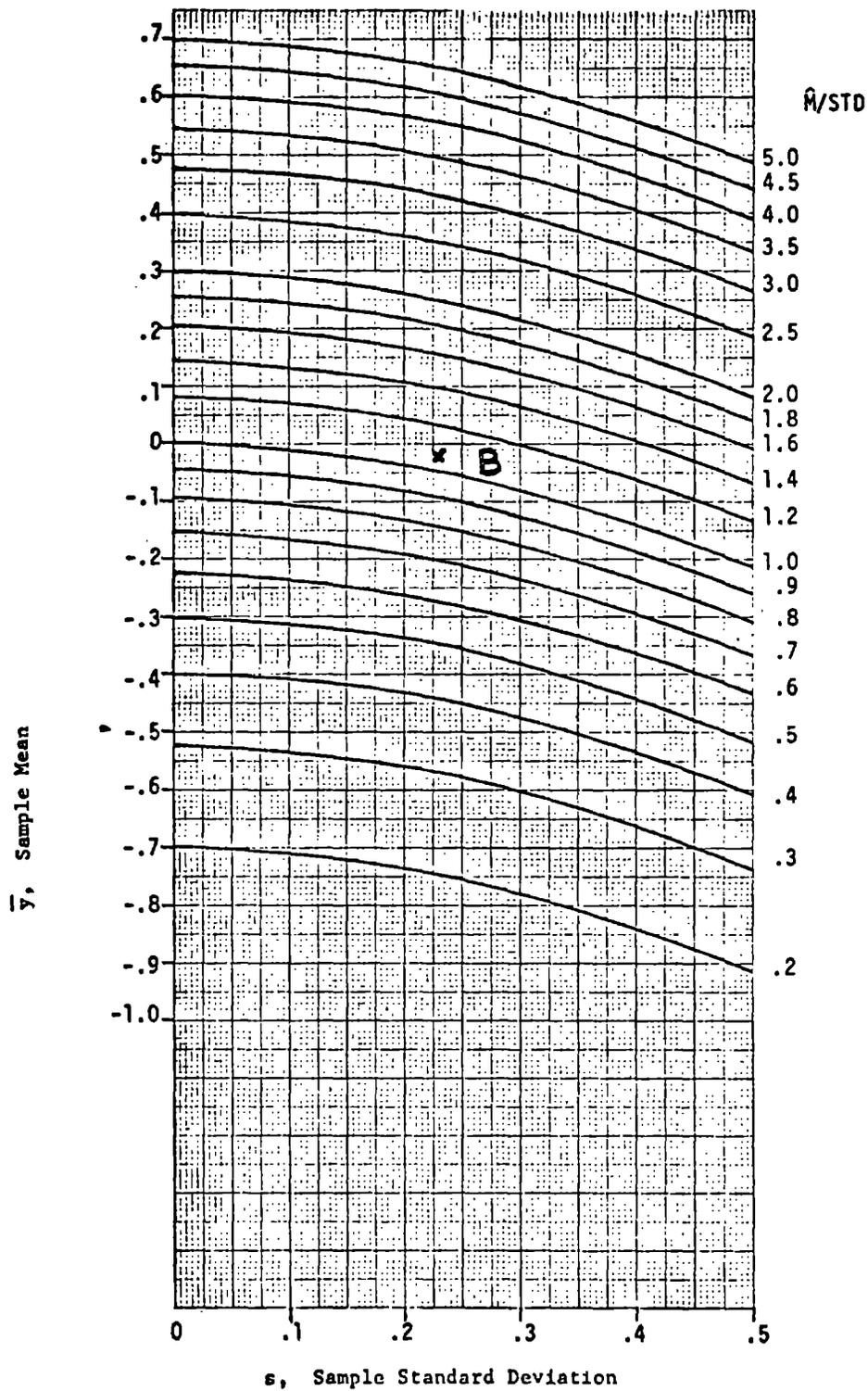


Figure 3.4 Estimation nomograph for Example #2 ($n = 6$)

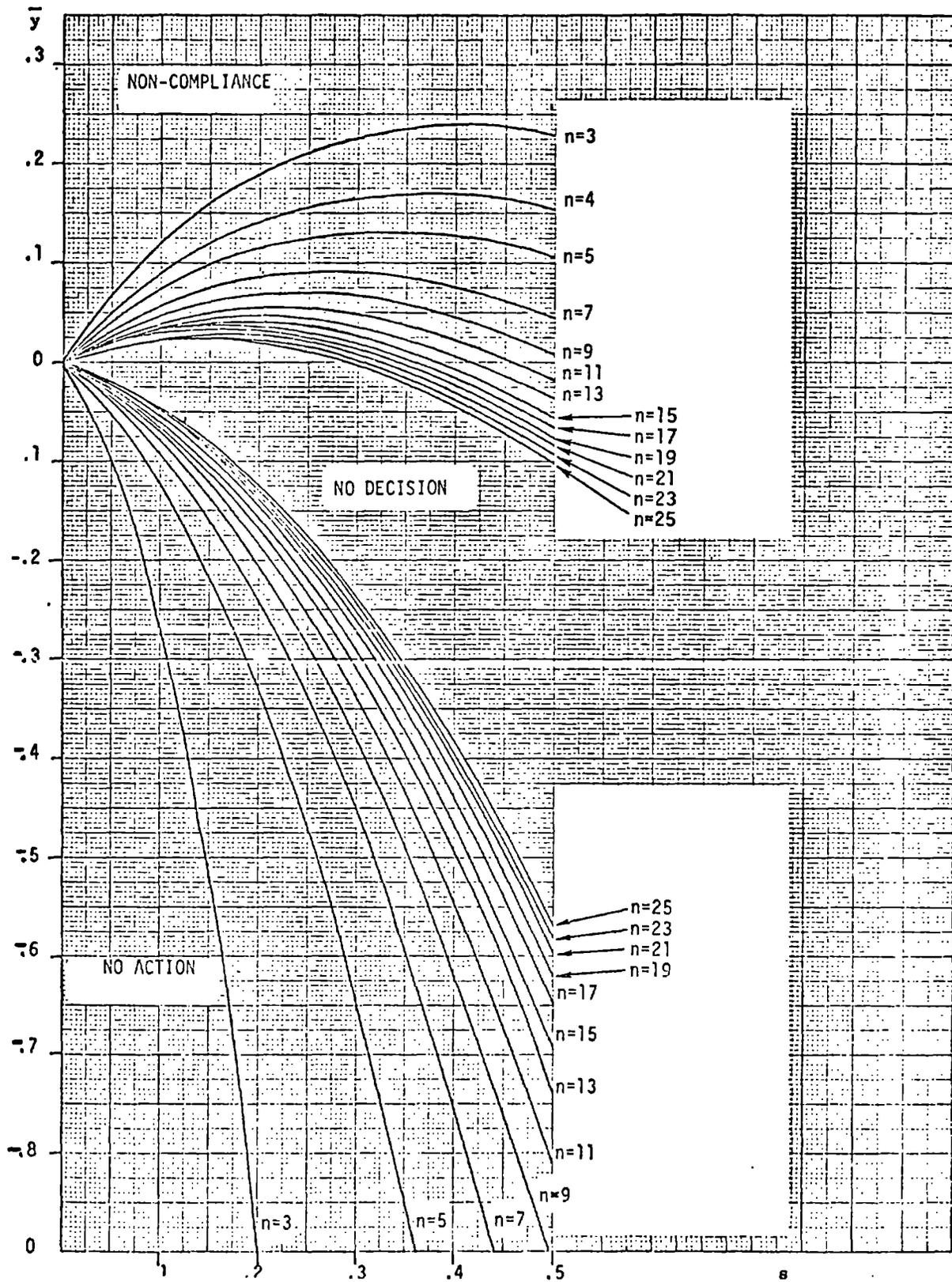


Figure 4.1 Decision regions

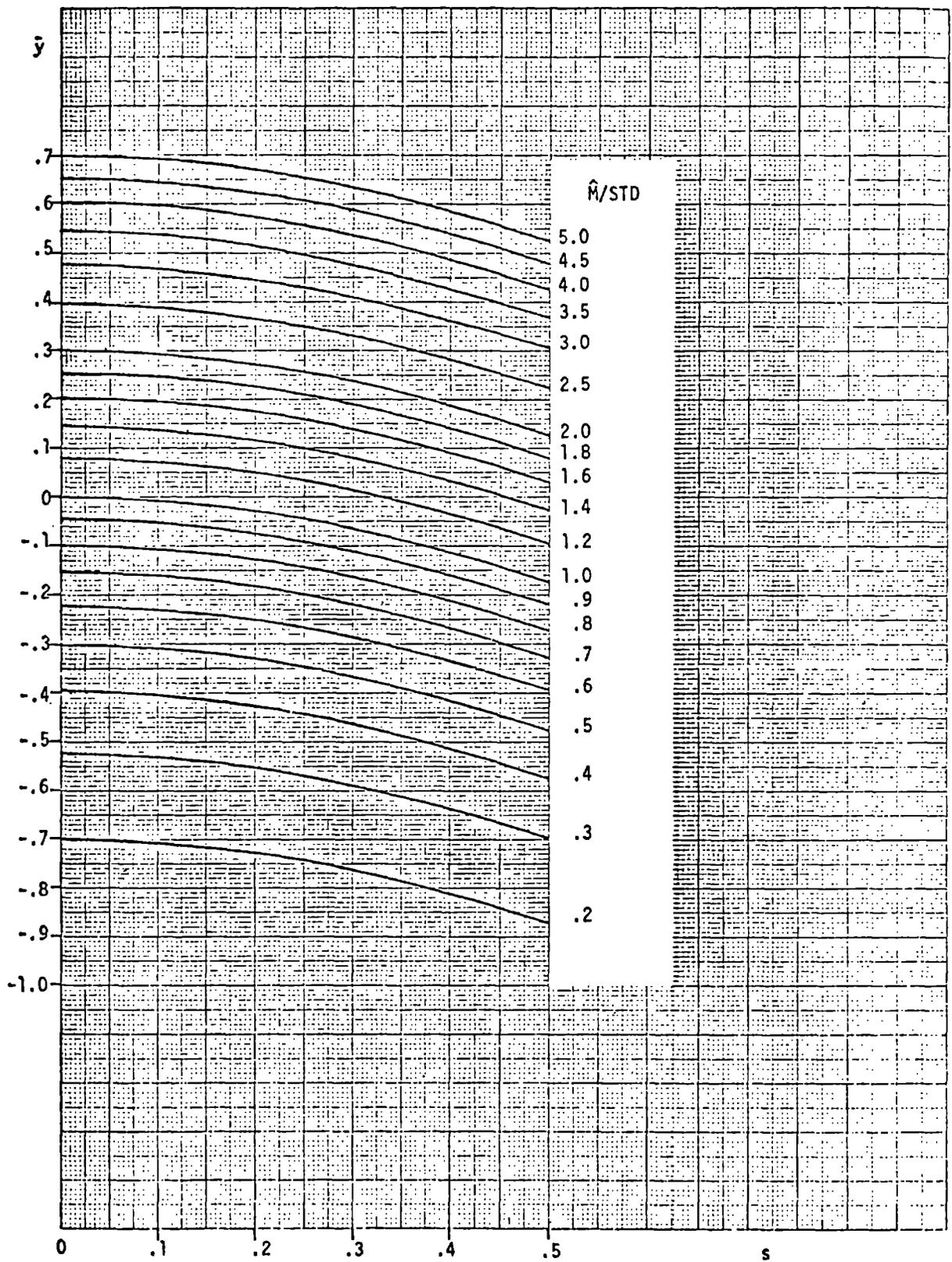


Figure 4.2 Estimation nomograph for $n=3$ or 4

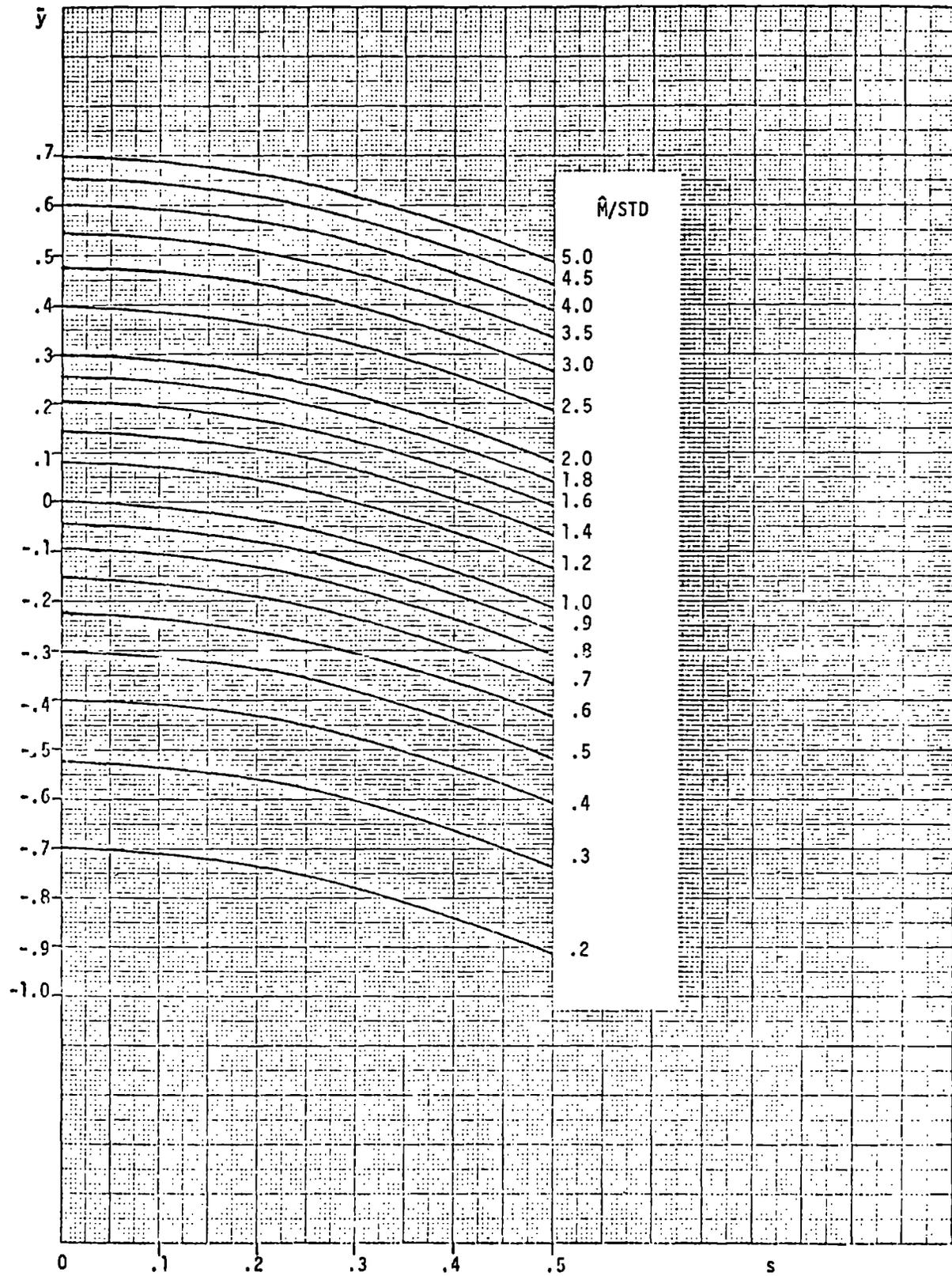


Figure 4.3 Estimation nomograph for $n=5$ or 6

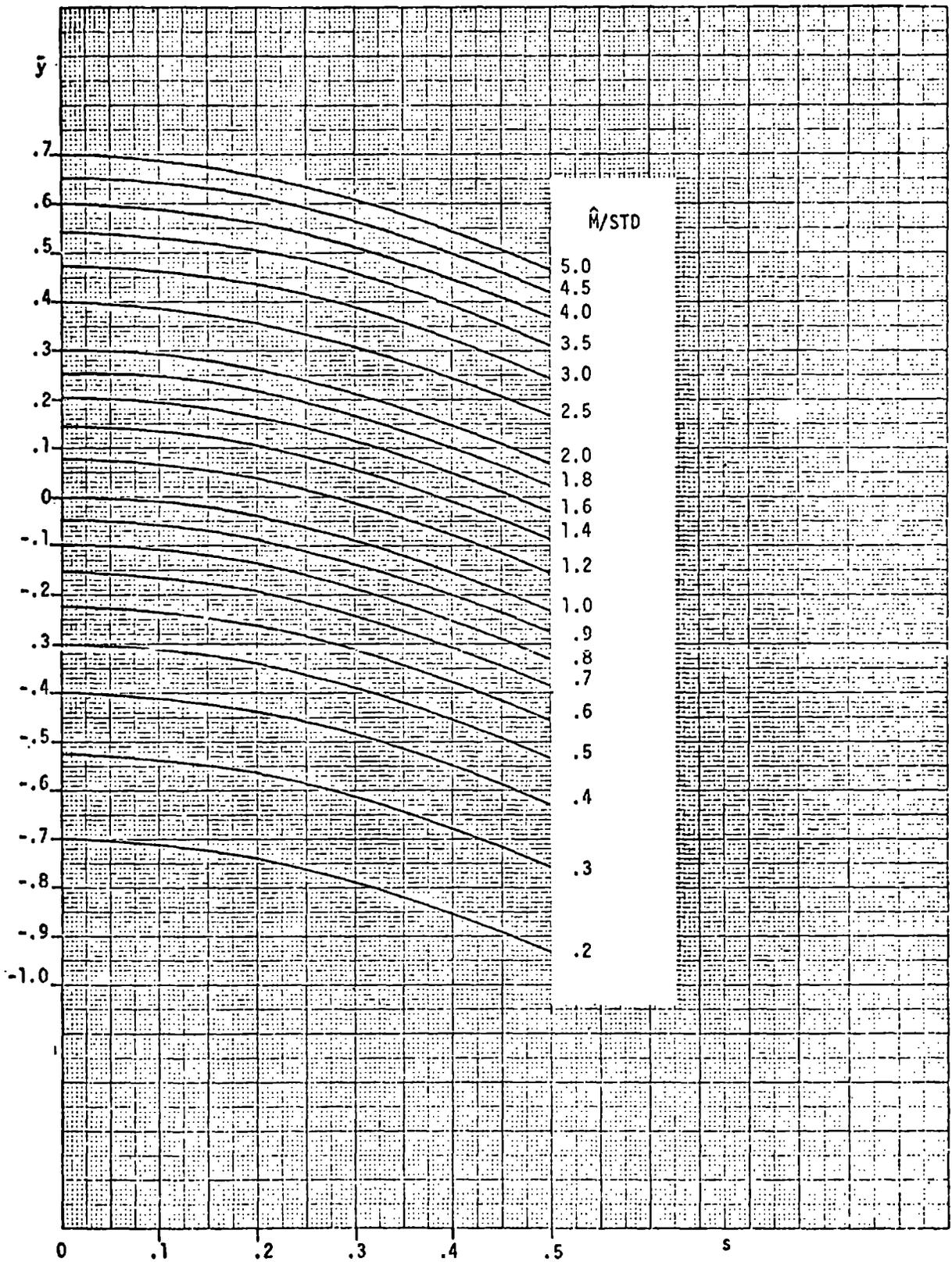


Figure 4.4 Estimation nomograph for $n=7$ or greater